TALLINN UNIVERSITY OF TECHNOLOGY
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VIPS - Visual problem solver

Master thesis

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Author declaration

I herewith declare that this is my own individual work and has not been proposed for defense anywhere before. All used in the thesis works, views and statements of other authors are properly referenced.

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Annotatsioon

Käesoleva töö peamiseks eesmärgiks oli luua veebipõhine programm, mis võimaldab kasutada visuaalset modelleerimise meetodit matemaatiliste, füüsikaliste, majanduslike jms probleemide lahendamiseks intuitiivsel ja interaktiivsel moel. Programm on peamiselt mõeldud kasutamiseks koolides või teistes asutustes õppimise eesmärgil.

Viimased trendid akadeemilises valdkonnas näitavad olulist kasvu mobiilse ja puutetundliku ekraaniga toodete nagu tahvelarvuti kasutusele võtmisest. Sellistel seadmetel on palju häid omadusi, mis aitavad õpetajaid ja õpilasi õppetöös. Olemasolev õppimise tarkvara aga ei tööta sellistel seadmetel, mis on osaliselt tingitud tehnilistest piirangutest ja muudest faktoritest. Nõudlus tahvelarvuti jaoks loodud õppetarkvara järele on olemas. Selles töös ma analüüsin olemasolevaid tarkvaralahendusi, nende eeliseid ja puuduseid ning pakun omapoolse lahenduse.

Antud töö lõpptulemusena loodi veebipõhine programm, mis annab õpetajatele ning õpilastele võimaluse omavahelist tööd paremini koordineerida ning lahendada ülesandeid kiiremini intuitiivsel ja tavapärasel moel. Programmi saab kasutada tahvelarvuti või muu seadme peal millel on olemas interneti ühendus ja HTML5 toetusega veebilehitseja.

Lõputöö on kirjutatud Inglise keeles ning sisaldab teksti 101 leheküljel, 6 peatükki, 44 joonist, 16 tabelit.

Abstract

The primary goal of this work was to create a browser-based program which promotes visual, model-based methods in solving problems in mathematics, physics, economics, etc in intuitive and interactive way. This program will be primary used by schools or other institutions for educational purposes.

One of the major trends in academics for the past few years has been a very fast transition towards mobile touch based devices, such as tablets. These kinds of devices provide many benefits for both teacher and pupil in educational work. However existing educational software hasn’t been ported to these platforms yet, partly due to limitations of platforms and other factors. There is a need for such educational software on tablets. In my work I will analyze existing solutions their advantages and disadvantages and propose my own solution.

As a result of this work a browser-based program was created to allow both teacher and pupil collaborate and solve exercises quicker in intuitive and natural way using tablet or any other device that has Internet connection and compatible HTML5 browser.

Master thesis is written in English, contains 101 pages, 6 chapters, 44 figures, and 16 tables.

Keywords

Virtual keyboard ***For devices that doesn’t have keyboards, virtual keyboard allows user to enter keys or other values, using on-screen keyboard.***

Virtuaalne klaviatuur - Seadmetele millel ei ole füüsilist klaviatuuri, virtuaalne klaviatuur annab võimaluse lisada sümboleid ekraanil olevast klaviatuurist.

Exercise template ***For solving exercises, teacher can create a template that contains information about the exercise problem and what pupil has to do to solve it.***

Ülesande mall – Ülesande lahendamiseks õpetaja saab luua malli kus on selgitatud ülesande püstitust ja mida õpilane peab tegema et see ülesanne lahendada.

Exercise strategy ***When solving exercise, there can be multiple ways how it can be solved; each separate method of solving exercise is called strategy.***

Ülesande lahendamise strateegia – Ülesannet saab lahendada mitut moodi, erinevaid lahendamisviise nimetakse strateegiaks.

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1 Introduction

1.1 Rationale

In this work I am implementing a browser-based program which promotes visual, model-based methods for solving problems in mathematics, physics, economics etc in intuitive and interactive way. Main target audiences for this program are school and pupils with growing use of tablets and phones for their class- and homework.

1.2 Trends in education and problems

In the following section an overview of changes in educational procedures is explained, such as increased importance of science teaching and changes in popular hardware that is used in schools by pupils. This establishes a need for a new browser based interactive program for developing problem-solving skills.

Many pupils and students today have to solve practical examples, but with a growing complexity of tasks, current mainly paper-and-pencil-based methods are not sufficient, so a new and better ways of teaching have to be developed, for example using data visualization. With a rise of popularity of tablet and other mobile devices, and rapid decline of PC market [1], many schools are starting to use tablets in educational purposes. However older applications that were developed for personal computers do not work on tablets or other new mobile devices. Due to the fact that many devices have different hardware and operating systems the only cross-platform way of developing such applications is to write them for Web Browser in HTML.

In Estonia, according to the latest government study program, Mathematics and close disciplines – programming, physics, chemistry and biology will be the most important courses for many years to come [2]. The target for the future is to prioritize fields such as engineering and technology, and here mathematics is the key part [2]. The requirements are very demanding and courses are difficult to study. Many teachers in smaller schools and pupils fear that they can’t get decent grades by the time when a final exam takes place [2].

Many applications for education already exist and new ones are getting created [3], but many of them only target specific device or operating system, and often are not compatible with other devices, such as tablets or mobile devices that are quickly gaining in popularity.

One possible way to write application once and use it later on many different kinds of devices, is to write it in HTML 5 and run it from compatible web browser for this device. Since HTML5 is relatively new technology, there are barely any educational applications written using this technology.

Teachers need new tools that can help them to be more productive, but we also need to rethink the whole idea about how the teaching is done in the first place.

For improving the educational process itself, recent shift [4] has been to change the way how mathematical assignments or tasks are solved. No longer is the goal to solve mathematical tasks and memorize equations, but to actually solve real-life problems, that people can encounter in the future [4].

1.3 Intended target audience

This program is intended for any kind of academic institution such as school or college, where simple mathematical equations are used for teaching courses like mathematics, chemistry or physics.

The modern learning process for studying mathematics requires pupil to have: books, text book, pen and calculator or sometimes a computer program if it is provided by school. Not much has changed in the hundreds of years since the first public schools were created. With the improvements in technology there are now far better ways to work with data and present information in intuitive and interactive way, especially for academic purposes, where this is extremely important.

1.4 Modern technology benefits

There are many improvements in both software and hardware that has happened in the past 10 years. The introduction of tablet devices and improvement of internet HTML standard now provide a powerful platform for developing next generation of Web applications for education.

1.4.1 Tablet devices

One new category of devices that has been out for the few years on the market is called tablets, and one of the most well-known is Apple's - iPad [5]

Figure 1 Apple iPad

iPad is a relatively small device with a touchscreen. It is running an Operating System (iOS), has rich multimedia features, thousands of downloadable applications and wireless internet connection [6].

Since it is extremely portable (241 x 18 x 9 cm in size) and very light (652 gr in weight) if we can compare it to the weight and size of a regular book, then this actually makes it a perfect device for academic use, and current trends show that many academics are excited about it [7].

According to the latest research, students in United States are already using tablets for their studies, and would like to use them even more [8]. Many academics believe that tablets are the future of learning [7].

There are many other tablets on the market but iPad is a preferred choice for academics at this moment, since it has been more widely adopted [7]

The Los Angeles Unified board has approved a 30 million $ contract to buy an iPad tablet for 30 000 students in 47 schools [9]. This iPad will come with pre-installed learning software and ability to use online courses for studying.

“Tablet Initiative” from Elon University also encourages students to use tablets for their studies [10].

There are many benefits for using mobile devices such as tablets for learning purposes:

- Portability – devices are more mobile when compared to notebooks or personal computers, so users can keep their notes in one single place, and take device with them almost anywhere [7].

- Far lighter and compact than many backpacks pupils carry that are filled with books and textbooks [11].

- Many users are already familiar with modern technologies, such as phones or tablets, and don’t need allot of time to get familiar with it [7].

- Encourages collaborative learning, sharing ideas, resources and interaction between pupils and teacher [7].

- Easier to access information through internet, conduct discussions and communication [7].

- A huge number of different applications that are available for different fields of study (physics, history, biology, science) [12]

1.4.2 Web Applications

While tablet itself is a hardware device, it is useless without software that is running on it. With modern technologies there has been a major shift in recent years towards Web Applications.

Web application is a computer program that is running on a device through Web Browser. It is different from traditional web page, because Web Application treat the browser as its own platform, independently from Operating System, but allowing many features of Operating System to be utilized, that regular web page cannot (Multiple threads, rich 2D / 3D graphics, fonts, data storage, etc). Until recently this would have not been possible without installing a custom Web Browser pluginis.

Traditionally software has been written for specific hardware or Operating System. Every Operating System is different, with different features and architecture choices, so the Software developer had to write so called cross-platform applications that could work in multiple Operating Systems without major changes. Sometimes software had to be written specifically for one Operating System, since it was not possible to write it any other way. This has been a major problem for Software developers for many decades.

Nowadays all operating systems are coming out with pre-installed Web Browsers. With introduction of HTML5 most major Web Browsers already have limited support for this standard [13]. This means that, by writing Web Application in HTLM5 software developer doesn’t have to worry about different hardware platforms or Operating Systems, and the Web Application will work the same way, even if it is opened on iPad, Linux or Windows PC.

This makes Web Applications an ideal choice to be used when writing application for tablets.

Benefits of Web Applications:

- Ease of access – installation is not needed; everything that user has to do, is to open specific web page [14].

- Ease of deployment – Most typical applications have auto-update features, so the application can retrieve and install latest version of itself. This is not needed for Web Application, since the application itself is on the Web location and it always has the latest version [14].

- Familiar user interface – Many users are already familiar how to use web browsers (navigate between pages, bookmark, submit buttons, etc.) [14].

- Native platform independence – Since applications are written for specific web technology like HTML5, the application does not depend on specific Operating System it is running on, and work regardless, as long as Web Browser supports HTML5 features that were used.

1.5 Overview of educational methods

Main focus of this section is to analyze what is a traditional approach taken when solving mathematical assignments and what are the main problems associated with this.

The most important aspect of mathematics is a problem solving [15]. The ability to solve mathematical problems requires the following operations: overview, testing, analysis, synthesis, generalization, summarization, using analogy, deductive thinking, etc [15].

Teachers commonly use the Pólya model for solving mathematical problems. This model consists of four steps [16]:

- Understand the problem
- Devise a plan to solve the problem
- Implement a solution plan
- Reflect on the problem

Many exercises in schools for mathematics, physics and other sciences follow this schema (see “Calculation examples”, 6.2).

One of the key elements that exercises commonly have (Step 1 in Pólya model):

- What is known – such as constants, known variables, etc.
- Visualization – drawing of the problem.
- What is the goal – how to solve this problem.

It is up to pupil to follow remaining steps until the goal is fulfilled.

Typically exercises are solved in class or at home as a homework assignment. Assignments are later verified by teacher if they were solved correctly or not.

While in class there is a teacher or classmates who can help, the situation is different when assignments are made as homework.

While Pólya model is an excellent way to solve tasks, there are many problems that pupil will encounter when working individually at home (this applies to assignments that are written in a regular test book with one or more answers for assignments as results for self-check):

- An assignment in test books typically shows only end results, but it doesn't show the actual way how assignment was solved. While this is a good choice for self-checking, it doesn't help pupil in situation where he can get stuck and doesn't know how to continue.

- Sometimes an assignment can be solved in different ways but end results can be the same. From teaching perspective it is important for pupil to find different ways how the same problem can be solved (part of Pólya solving model, step 2). This is beneficial even in those cases where pupil can get wrong results in the end, as this improves his solving technique by trying different approaches [15].

- There is no other help except than a book, course notes, or computer software that can help him in situation where he doesn't know how to proceed. These all require allot of effort and time to find an answer and in the end may cause him to get frustrated and not finish homework at all.

1.6 Main goals of this work

Based on analysis of existing problems and available technologies, the following list is created and represents the main goals of this work:

1. Create requirements for the program

2. Analyze other alternative solutions based on the requirements

3. Implement visual solver application

1.7 General requirements for program

Based on the declared goals in section "Main goals of this work",1.6 and considering problems both in technical, educational aspects and benefits that modern technologies provide with mobile device (tablets, phones) the following list of requirements is created.

Definitions:

- User – either pupil or teacher. They have different roles (teacher creates assignments, pupil solves them)

- Exercise – assignment (mathematical problem) that pupil have to solve. One assignment can contain one or more ways how to solve it. Each way includes one or more solving steps that include equation or text describing an action that is done at that moment.

- Strategy – A method of solving exercise. For a single exercise multiple strategies can be created, while they differ in steps taken, the end result is the same.

- Score – A numeric value indicating how well assignment was completed by the pupil. Pupil can request help from program when it is needed, but every time help is used, the final score is decreased. A perfect score is achieved when problem is solved without using any help. This can be used to motivate pupil to try again, improve the results and research for other ways how to get a better score. This method of motivational approach is called Gamification[1].

- Automatic evaluation – By default equations are evaluated, in order to save time for the pupil and help him focus on the creating exercise model itself. This means that pupil doesn't have to use electronic calculator for solving exercises.

The requirements are divided into different categories:

- Collaboration

- Visualization

- Interactivity

- Support

- Technical – assuming Internet connection is available.

[1] The process of game-thinking and game mechanics to engage users and solve problems [38].

Requirements for a program:

Nr	**Category**	**Description**
1.1	Collaboration	Teacher can create an exercise and assign it to pupils.
1.2	Collaboration	Pupil can solve exercise and share solution with teacher for verification of results.
1.3	Collaboration	When pupil solves the exercise a program will evaluate automatically if it was solved, provide an option to display alternative solution.
2.1	Visualization	Mathematical equations should be written and displayed naturally as they appear in original source book (without using complicated names specific only to this program, or require a manual to read).
2.2	Visualization	For an exercise a picture explaining the problem can be added, displayed or removed
3.1	Interactivity	Multiple solutions can be added for a single exercise even with different end results. Every solution includes full steps from beginning till the end.
3.2	Interactivity	Written equations will be evaluated automatically.
3.3	Interactivity	Problem solving steps are shown, each containing one or multiple equations. Last step displays end results. User can switch and move between steps and make modifications.
3.4	Interactivity	Evaluated equation, show results of evaluation and also display current state of any values that were previously evaluated
4.1	Support	A list of known formulas is available to any user and can be modified if required

4.2	Support	If pupil is stuck with an exercise and cannot continue, an option is shown with possible evaluation steps or a list of formulas that can be used for evaluation. Using any option will deduct final score, but allow pupil to continue
4.3	Technical	Program must work primarily on a tablet device, and other devices if possible
4.4	Technical	Open source – Source code should be open, using industry standard and free technologies without restrictive licensing, so it can be changed by the teacher/school and additional functionality can be added.

Table 1 General program requirements

1.8 Analysis of existing solutions

There are many programs on the market, but many of them are not working on tablet devices or not available on the web. In this section I am going to define requirements for analysis and compare many popular mathematical software solutions on the market.

1.8.1 Methodology of comparison of mathematical solvers

Based on general requirements described in section 1.7, the testing scenarios are created for every solution under test.

Test scenario nr	**Tested requirement**	**Condition to test**
1	1.1	Every exercise is created as a separate entity from other exercises
2	1.1	One exercise assignment can be assigned to many pupils
3	1.2	Teacher should be able to track status of assignment (if it was completed by the pupil)

4	1.2	Changes made by the pupil in exercise should be visible to teacher.
5	1.3	If exercise is solved by the pupil, the program should be able to notify pupil if results he has obtained are correct
6	2.1	Mathematical equations should be written and displayed naturally as they appear in original source book (without using complicated names specific only to this program, or require a manual).
7	2.2	For an exercise a picture explaining the problem can be added, displayed or removed
8	3.1	Every exercise can have multiple solutions added by the pupil, so the teacher can review them later.
9	3.2	When equations are written they will be calculated automatically
10	3.3	Exercise should be able to have non-fixed number of steps for solving exercise
11	3.3	Every step can be separately added/deleted or modified by the user
12	3.4	Evaluated equation, shows results of evaluation and also displays current state of any values that were previously evaluated
13	4.1	A list of known formulas is available when working on exercise
14	4.1	Teacher can add formulas to the list or modify them
15	4.2	If pupil is stuck, he can request help and unlock parts of the exercise solution that was previously made by the

		teacher
16	4.3	Program must be working on a tablet device
17	4.4	Program should be licensed with Open Source Software license, so the code can be manually changed depending on the requirements of the school.

Table 2 Test scenarios for solvers

Testing of the following scenarios is performed with exercise in section 6.2.1

1.8.2 List of mathematical solvers for comparison

Here is a list of some of the most widely used applications for solving and calculating mathematical equations. They are very different in features and technologies used:

Nr	**Name**	**Description**	**Free**
1	Microsoft Excel 2010	Discover and reveal the insights hidden in your data [17]	No
2	Maple 17	The result of over 30 years of cutting-edge research and development, Maple combines the world’s most powerful mathematical computation engine with an intuitive, “clickable” user interface. [18]	No
3	Wolfram Mathematica 9	*Mathematica* is renowned as the world's ultimate application for computations. But it's much more—it's the only development platform fully integrating computation into complete workflows, moving you	No

		seamlessly from initial ideas all the way to deployed individual or enterprise solutions. [19]	
4	MathWay	Mathway provides students with the tools they need to understand and solve their math problems. With hundreds of millions of problems already solved, Mathway is the #1 problem solving resource available for students, parents, and teachers. [20]	Partial
4	Algebra Solver and Math Simplifier	Graphical Universal Mathematical Expression Simplifier and Algebra Solver (GUMESS). It solves most middle school algebra equations and simplifies expressions and it SHOWS ALL WORK. [21]	Yes
5	T-algebra	T-algebra is designed for helping in learning algebraic equations for 4-8 grade schools [22]	Yes
6	Scilab	Scilab is free and open source software for numerical computation providing a powerful computing environment for engineering and scientific applications. [23]	Yes

Table 3 List of available mathematical equation solvers

1.8.3 Comparison results

The following section is created based on “Main goals of this work”, 1.6 and contains a list of comparisons between different mathematical solvers with results that were obtained. Comparison is made in order to analyze existing solutions and understand what kind of features worked best and if some of them can be used instead.

1.8.3.1 Microsoft Excel

Tests were performed on Microsoft Excel 2010

Test scenario nr	**Passed**	**Results**
1	YES	Every exercise can be created as a separate Excel file
2	YES	There are many options to share excel files: Send files with email Save file to Microsoft Sky Drive Save file to Sharepoint
3	NO	There is no automatic way to track progress. Once Excel file is created, it has to opened again to see results.
4	Partial	There is no built-in way to do this. One possible option is to use SharePoint or Sky Drive, but this has to be agreed on between teacher and pupil first.
5	NO	This requires allot of effort from teacher side, and will make exercise details visible for some pupils.
6	NO	Equations as other Office suite applications have an option to insert equations, but they are not used in calculations directly.
7	YES	Using Insert option from menu
8	YES	There are no limitations what can be added, but it is not conveniently implemented.
9	NO	Equations are not calculated automatically, user has to write the formula manually once and only after that, if values change the result will be evaluated. If formula needs to be changed, user has to rewrite it in excel manually again.

10	YES	There is no such thing. User is free to add any number of formulas and calculations and call them steps, it is not convenient to use though.
11	YES	There is no such thing. User is free to add any number of formulas and calculations and call them steps, it is not convenient to use though.
12	YES	This can be done, but require extra work for the user by adding these values manually.
13	NO	No such thing built-in. There is a list of equations, but it is not the same thing, because it cannot be used in calculations.
14	NO	No such thing built-in. There is a list of equations, but it is not the same thing, because it cannot be used in calculations.
15	NO	No such thing available.
16	N/A	There is a version of Microsoft Office specifically created for tablet devices (https://itunes.apple.com/us/app/office-mobile-for-office-365/id541164041?mt=8) However it requires a separate license and subscription than retail version. The tests were performed on desktop version instead.
17	NO	This is proprietary closed software.

Table 4 Testing results for Microsoft Excel

Excel is a very powerful tool that is capable of performing all kind of calculations, statistical operations, etc. It has strong sharing capabilities, and plenty of choices to perform efficient calculations once the mathematical model is created. For this specific project, Excel lacks capabilities to create dynamical models in real-time and calculate equations as they are added on the fly by the user.

1.8.3.2 Maple 17

Testing was not performed. Demo version was not available for evaluation.

1.8.3.3 Wolfram Mathematica 9

For these tests Wolfram Mathematica 9.0.1 was used.

Test scenario nr	**Passed**	**Results**
1	YES	Mathematica operates with notebooks, single notebook can contain pictures, text, expressions, equations, functions, plots, graphs, etc
2	NO	Except "Send to", there is no other sharing option. The notebook content can be embedded in the web page, but it can be used for viewing purposes with some limited interaction.
3	NO	Without sharing, there is no way to track progress.
4	NO	Pupil has to manually send notebook to teacher.
5	NO	No way of doing this right now.
6	YES	There is an excellent support for displaying math in natural way.
7	YES	Picture can be added.
8	YES	Notebook can have multiple sections with different evaluations.
9	NO	Calculations are not done automatically; user has to type SHIFT-ENTER in the current cell for the values to be evaluated.
10	YES	Number of steps or (cells in this case) has no restriction.

11	YES	
12	YES	When evaluating all of the notebook
13	Partial	Teacher can create a list of functions or formulas that can be used, but only for this notebook.
14	Partial	Teacher can create a list of functions or formulas that can be used, but only for this notebook.
15	NO	It was not designed for this purpose.
16	NO	Program is available on Windows, Mac and Linux.
17	NO	This is proprietary closed software.

Table 5 Testing results for Wolfram Mathematica

Wolfram Mathematica is an excellent program for performing mathematical computations and modelling of data, it doesn't however provide any of the required features for collaboration between people, and doesn't have support for tablet devices.

1.8.3.4 MathWay

Since MathWay is web based software, there is no clear indication what version was used. As a reference: testing was done in December 2013.

Test scenario nr	**Passed**	**Results**
1	YES	For MathWay, the exercises are called problems and each of them is listed separately
2	YES	Choosing sharing options of the problem allow exercises to be shared via: Direct URL Facebook, Tweeter

		Email
3	YES	Exercise can be shared with teacher, but it is not convenient, since there is no single list of all the pupils with assignments.
4	YES	Since it is web-based solution, changes are available instantly.
5	NO	Program can only solve the problem but only teacher can verify if it was solved correctly.
6	YES	The web interface has a good number of features and can show math in natural form. Virtual keyboard is also available for touch based devices.
7	NO	No such thing is available.
8	NO	It can be done, but in this case user has to share multiple links, not practical.
9	NO	Equations or expressions are only calculated when user clicks a button.
10	YES	There are no clear steps, every line of text can be considered a step. There is no defined limit for the number of lines.
11	Partial	Lines of text can be edited, but currently it is working inconsistently, cursor is shown in one location, but text is deleted in another. Lines can only be inserted to the bottom of the page.
12	Partial	Results of evaluations are shown, but variables themselves are not separately displayed.
13	No	Not supported.

14	No	Not supported.
15	No	Not supported.
16	YES	All features are working.
17	NO	Source code is available (due to JavaScript), but license is not specified indicating it cannot be modified without permission.

Table 6 Testing results for MathWay

When MathWay is used on a mobile device it provides a great value with good collaboration options, math that is written in natural way, exercises that can be shared in many ways, however it is not very well suited for the kind of solution that is required for this project.

1.8.3.5 Algebra Solver and Math Simplifier

Since GUMESS is web based software, there is no clear indication what version was used. As a reference: testing was done in December 2013.

Test scenario nr	**Passed**	**Results**
1	NO	Exercises can’t be created.
2	NO	Exercises can’t be created.
3	NO	Exercises can’t be created.
4	NO	Exercises can’t be created.
5	NO	Exercises can’t be created.
6	NO	The math is written in unnatural way, once it is parsed, the natural representation will be created and shown as an image.
7	NO	Not supported.

8	NO	Not supported.
9	NO	User has to press a button for evaluation to occur.
10	NO	No steps available.
11	NO	No steps available.
12	NO	Only one equation/expression can be solved/evaluated at a time
13	NO	Not supported.
14	NO	Not supported.
15	NO	Not supported.
16	YES	
17	NO	Source code is available (due to JavaScript), but license is not specified indicating it cannot be modified without permission.

Table 7 Testing results for GUMESS

GUEMESS completely doesn’t fit the requirements for the solution, since it was created for different purposes in mind.

1.8.3.6 T-algebra

For testing T-algebra version 1.1 was used.

Test scenario nr	**Passed**	**Results**
1	YES	Exercises are created as separate files; this can be used to group them according to the course topic.
2	YES	File can be exchanged with other pupils using physical means, such as email, flash drive. There is no built-in

		share or collaboration method.
3	YES	There is built-in method for this, including number of mistakes makes and number of times help was used.
4	YES	Changes made by the pupil in exercise should be available to teacher.
5	YES	Results are checked in the end and user is notified if there are issues.
6	YES	The editor is fairly easy to use and virtual keyboard is also shown when adding special symbols
7	NO	Not supported.
8	NO	No, only one solution is calculated by program automatically
9	NO	User has to specify the action that will be used and click on the button to perform this action.
10	NO	Number of steps is fixed and is pre-defined when exercise is created.
11	NO	Number of steps is fixed, so steps can only show correct values and cannot be changed to anything else.
12	NO	Only latest evaluation result is shown
13	NO	Not supported.
14	NO	Not supported.
15	YES	There is a way to request a help tip, for those cases when pupil doesn’t know how to continue.
16	NO	Only Windows platform is supported.

17	NO	Code is not available.

Table 8 Testing results for T-Algebra

T-Algebra is an excellent program for teaching pupils simple algebra. It works very well for education purposes, allows teacher to get a list of exercises that were completed and statistics of completion results. The biggest problem with T-Algebra, is that it works only on Windows platform and doesn't support tablet devices that are becoming allot more popular now.

1.8.3.7 Scilab

For the following tests Scilab 5.4.1 was used.

Test scenario nr	Passed	Results
1	YES	Exercise can be saved separately as a file.
2	YES	Created exercise can be shared with pupils and executed later.
3	NO	There is no built-in way for the teacher to track assignment progress.
4	NO	There is no built-in way to do so. Files should be exchanged manually in this case.
5	NO	There is no way to do this, unless manually writing methods that will do this.
6	NO	Expressions are written as normal text.
7	NO	Not supported.
8	YES	There can be multiple solutions but this is not something that is natively supported by the program.
9	NO	This is done manually by the user.

10	YES	There can be any number of commands added.
11	YES	Any command can be added or edited, since it is written as a text.
12	YES	Scilab contains a list of variables and their values.
13	Partial	Teacher can create methods, but it is done manually
14	Partial	Teacher can create methods, but it is done manually
15	NO	Not supported.
16	NO	Scilab is supported only on Linux, Windows, Mac OS X.
17	YES	Created under GPL license.

Table 9 Testing results for Scilab

Scilab is free and powerful software that allows users to perform complex calculations. It lacks collaboration features and requires users to learn custom syntax in order to solve complex calculations. Expressions are not shown in natural form.

1.8.4 Analysis of comparison results

From the test results it is clear that T-Algebra is the best program based on the requirements. This can be explained because T-Algebra was primarily designed for educational purposes in the first place; so many requirements are similar as well. Since times has changed and so did the devices that are used today, the program became slightly outdated (last version was released 2 years ago)

Other software programs are not designed primarily for education purposes, but are designed to be either flexible or good at all things at once. Many of the programs are also bound to specific platforms.

By designing software for specific purpose only, the efficiency of the software to solve these tasks is increased so it becomes much easier to use and learn. T-Algebra is a good example in this case.

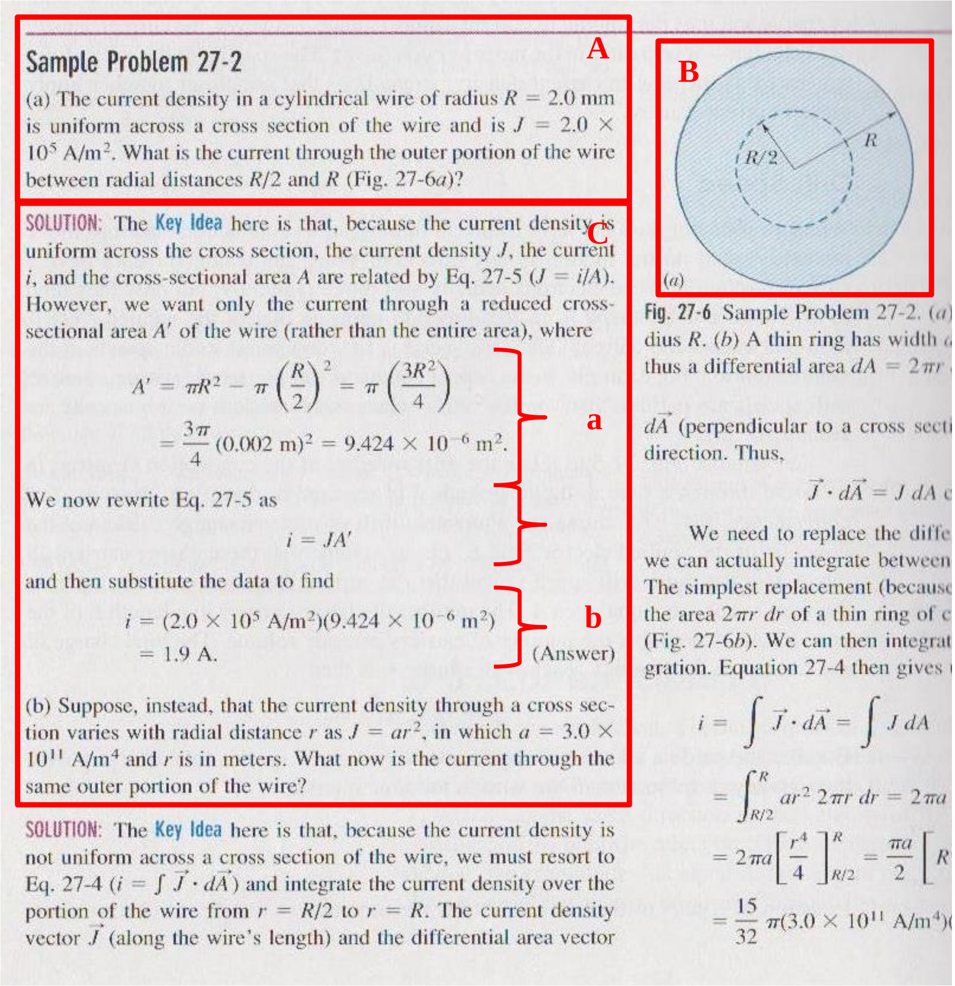
The biggest problem with T-Algebra is a lack of collaboration options and limitation of software that can work only on Windows platform.

1.9 Solution to be implemented

In this section I will analyze what is the best way to solve the problems mentioned, and how my solution will be implemented.

1.9.1 Software design goals and rationale

In the following part I will explain software design decisions that were made and reasoning behind it. Since main users of this program will be teachers, students or pupils, we have to understand that many of them are familiar with writing assignments in their workbooks, reading technical text in scientific books or papers, so the visual representation of the exercise should be as close as possible to the original source of information.



Sample Problem 27-2

(a) The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5$ A/m^2. What is the current through the outer portion of the wire between radial distances $R/2$ and R (Fig. 27-6*a*)?

SOLUTION: The **Key Idea** here is that, because the current density is uniform across the cross section, the current density J, the current i, and the cross-sectional area A are related by Eq. 27-5 ($J = i/A$). However, we want only the current through a reduced cross-sectional area A' of the wire (rather than the entire area), where

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \pi\left(\frac{3R^2}{4}\right)$$
$$= \frac{3\pi}{4}(0.002 \text{ m})^2 = 9.424 \times 10^{-6} \text{ m}^2.$$

We now rewrite Eq. 27-5 as

$$i = JA'$$

and then substitute the data to find

$$i = (2.0 \times 10^5 \text{ A/m}^2)(9.424 \times 10^{-6} \text{ m}^2)$$
$$= 1.9 \text{ A}. \qquad \text{(Answer)}$$

(b) Suppose, instead, that the current density through a cross section varies with radial distance r as $J = ar^2$, in which $a = 3.0 \times 10^{11}$ A/m^4 and r is in meters. What now is the current through the same outer portion of the wire?

SOLUTION: The **Key Idea** here is that, because the current density is not uniform across a cross section of the wire, we must resort to Eq. 27-4 ($i = \int \vec{J} \cdot d\vec{A}$) and integrate the current density over the portion of the wire from $r = R/2$ to $r = R$. The current density vector $\vec{J}$ (along the wire's length) and the differential area vector

Fig. 27-6 Sample Problem 27-2. (*a*)
dius R. (*b*) A thin ring has width
thus a differential area $dA = 2\pi r$

$d\vec{A}$ (perpendicular to a cross secti
direction. Thus,

$$\vec{J} \cdot d\vec{A} = J\, dA$$

We need to replace the diffe
we can actually integrate between
The simplest replacement (becaus
the area $2\pi r\, dr$ of a thin ring of
(Fig. 27-6*b*). We can then integrat
gration. Equation 27-4 then gives

$$i = \int \vec{J} \cdot d\vec{A} = \int J\, dA$$
$$= \int_{R/2}^{R} ar^2\, 2\pi r\, dr = 2\pi a$$
$$= 2\pi a\left[\frac{r^4}{4}\right]_{R/2}^{R} = \frac{\pi a}{2}\Big[R$$
$$= \frac{15}{32}\pi(3.0 \times 10^{11} \text{ A/m}^4)($$

Figure 2 Example of assignment (Scanned from book [24])

Let’s take a closer look at the Figure 2 Example of assignment (Scanned from book) and analyze the highlighted parts.

A. In the beginning of every assignment there is a short text that includes:

    a) Description of the problem

    b) Known values

    c) What needs to be found (typically value to evaluate)

B. An image to visualize the assignment problem better

C. A solution for the assignment that includes:

    a) One or many steps with mathematical equations and substitution operations
    b) Evaluated result that was required to be found

Many people are familiar with this approach of writing assignments (see previously mentioned Pólya solving model in section 1.5). My program will follow this approach as well and include all of the mentioned elements, not only that, but the mathematical formulas or equations (seen in Figure 2 Example of assignment (Scanned from book)) are also going to be displayed the same way as they are shown in the book.

Many people use electronic calculators today to solve these kinds of assignments, but calculators are tricky to use. They are very powerful, but complex (require user to read manual) and expensive in price for more feature rich models. It takes too many button presses to do even simple actions and making a mistake can cause the whole calculation to fail. This will require user to start all over again.

For my program, I will write my own solver, which will be used to solve algebraic expressions or equations if necessary values are known. The solver will take a natural form of mathematic expression and doesn’t require user to read manuals to operate or remembering custom syntax. It will be completely natural for the user. It does come with some limitations, since it is not possible to implement many complex mathematical operations and evaluate them in real-time, only a limited number of most commonly used operations will be supported. To be specific, -solver should support at least some basic operations that most calculators have.

In summary, the assignments for the pupil that will be created with this program should resemble the same assignments in typical workbooks that are currently used. The calculations will be made by the program itself, so there will be no need to use external calculators.

1.9.2 Software development method

In this section I will be describing software development process and other methods that will be used for implementing this program

For the development of the software program I decided to use traditional waterfall model. The waterfall model consists of the following steps:

1. Requirements and specification
2. Design and software architecture
3. Development and integration
4. Testing and debugging
5. Maintenance

The decision to use waterfall model instead of other models (such as using agile development process) was based on the fact that requirements are clearly defined, and not going to be changed much during development process, in this case development is going to be straight-forward.

1.9.3 Main actors

The program has only two actors:

Actor name	**Description**
Teacher	Teachers are employed by the school, teaching certain subjects such as mathematics, physics, etc. Teachers create assignments for the Pupils to solve, verify the assignment results and mark them with a

	grade.
Pupil	Pupils are studying in a school and taking certain classes (i.e. mathematics, physics, etc). Pupils receive assignments from teacher and solve them.

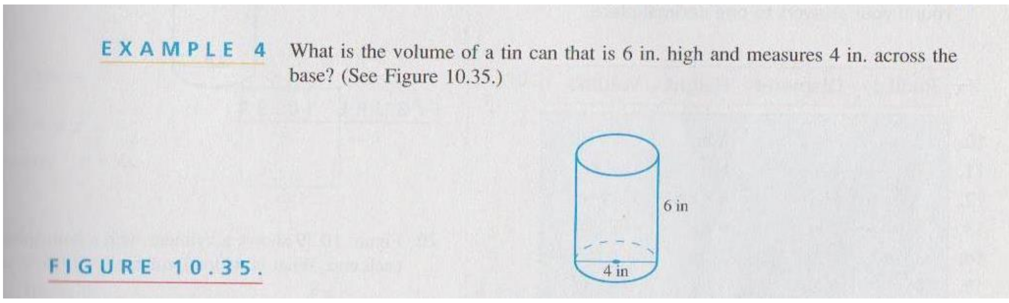
Table 10 Program actors

1.9.4 Definition of application entities

The following section describes naming of the entities that are used by the program.

Names are based on a list of definitions in section 1.7 and analysis in section 1.9.1, but are specific to the application specification and architecture, that's why they are described separately.

1. **Exercise template** – describes assignment details. Think of this as an assignment description that has to be solved (text with picture):

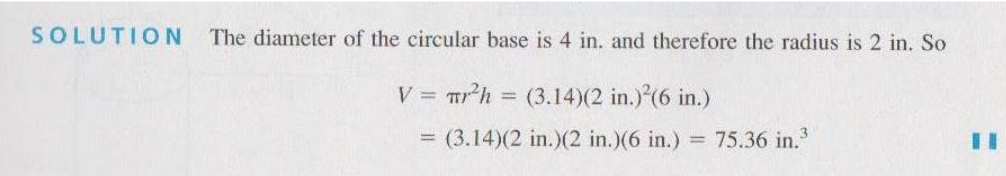


EXAMPLE 4 What is the volume of a tin can that is 6 in. high and measures 4 in. across the base? (See Figure 10.35.)

FIGURE 10.35.

Figure 3 Assignment details (Scanned from book [25])

2. **Exercise** - the solution to the exercise template (above). This solution typically consists of steps that were made in order to get required result:



SOLUTION The diameter of the circular base is 4 in. and therefore the radius is 2 in. So

$$V = \pi r^2 h = (3.14)(2\text{ in.})^2(6\text{ in.})$$
$$= (3.14)(2\text{ in.})(2\text{ in.})(6\text{ in.}) = 75.36\text{ in.}^3$$

Figure 4 Assignment solution (Scanned from book [25])

Both teacher and pupil can create exercises, but only the exercise that is created by the teacher is considered a solution.

3. **Exercise solution** – Exercise that was created by teacher. It is assumed that it is correct and can be used for the pupil to verify correctness of his exercise.

4. **Exercise strategy** – Every exercise can have multiple strategies. Strategy is a method of solving specific assignment, since there can be multiple ways how to solve one task, there can be multiple strategies as well.

5. **Exercise step** – Every exercise strategy can consist of multiple steps; every step is an action with an outcome, such as calculation, substitution of expressions or equations. In many cases it is not possible to solve assignment in one step, so they are usually divided into many steps, each performing specific operation.

Based on the requirements in section 1.7, the following use case diagram is created, displaying possible use cases and actors activity.

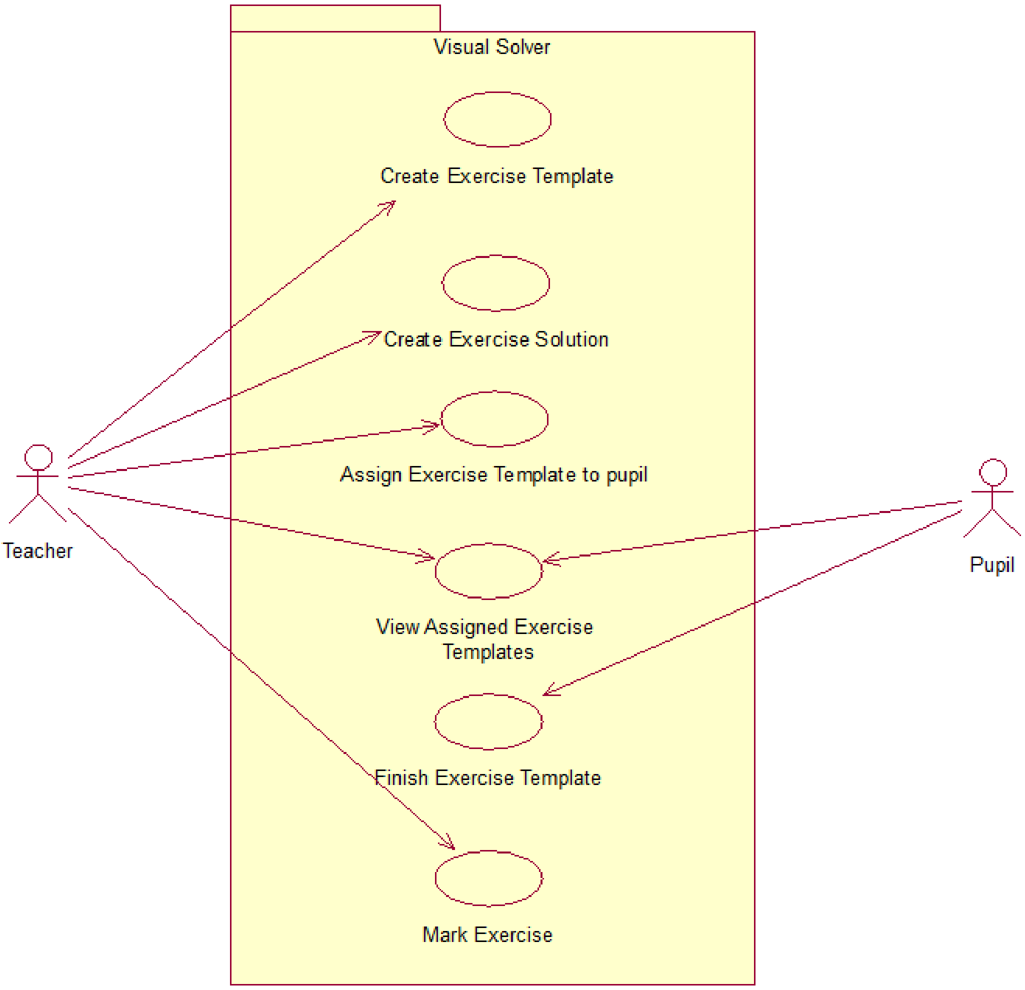


Figure 5 General use case diagram

Description of use cases is provided below:

Use Case ID	**1**
Use Case name	Create Exercise Template
Actors	Teacher
Goal	Exercise templates define what needs to be found and how it needs to be completed. The goal is to assign this exercise template to pupil and verify that pupil has knowledge of the exercise topic.
Pre-Condition	-
Post-Condition	Exercise template is created

Use Case ID	**2**
Use Case name	Create Exercise Solution
Actors	Teacher
Goal	Solution is an exercise created by the teacher for Exercise template. The goal for the teacher is to create an example for the pupil and explain how to correctly solve this exercise
Pre-Condition	Exercise Template is created
Post-Condition	Exercise Solution is created

Use Case ID	**3**
Use Case name	Assign Exercise Template to pupil
Actors	Teacher
Goal	Exercise Templates are assigned to pupils, so pupils can then create new exercise based on the template, and solve the assignment.
Pre-Condition	Exercise Template is created
Post-Condition	Exercise Template is assigned to pupil

Use Case ID	**4**
Use Case name	View Assigned Exercise Templates
Actors	Teacher, Pupil
Goal	The goal is to see what currently assigned exercise templates are, and what their status is (started, completed and marked).
Pre-Condition	-
Post-Condition	List of assigned exercises templates is shown

Use Case ID	**5**
Use Case name	Finish Exercise Template
Actors	Pupil
Goal	The goal for the pupil is to create exercise and finish it by marking it as completed.
Pre-Condition	Exercise Template is assigned to this pupil
Post-Condition	Exercise Template is finished

Use Case ID	**6**
Use Case name	Mark Exercise Template
Actors	Teacher
Goal	The goal for the teacher is to verify that exercise template was completed correctly by the pupil, and set mark accordingly, based on the correctness of the work that was performed.
Pre-Condition	Exercise Template is assigned to this pupil (can be finished or not)
Post-Condition	Exercise Template is marked

Both “Create Exercise Solution” and “Finish Exercise Template” use cases are based on “Create Exercise” use case, with slight modifications.

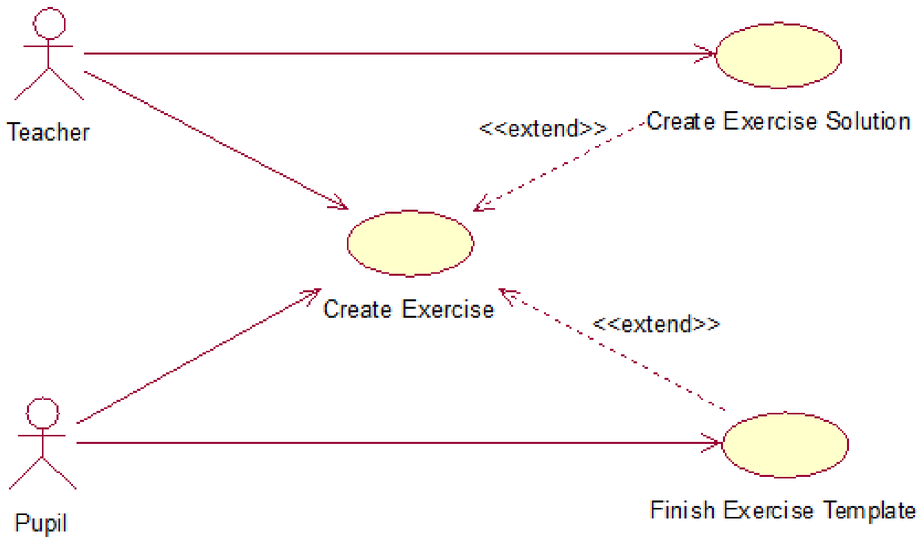


Figure 6 "Create Exercise" use case extension relationship

Use Case ID	**7**
Use Case name	Create Exercise
Actors	Teacher, Pupil
Goal	For every exercise template an exercise is created. The goal for creating exercise is to write down steps that are required for solving the assignment.
Pre-Condition	Exercise Template is available
Post-Condition	Exercise is created for Exercise Template

1.9.5 Actors activity diagrams

The following activity diagrams show in which order use cases are performed by teacher and pupil.

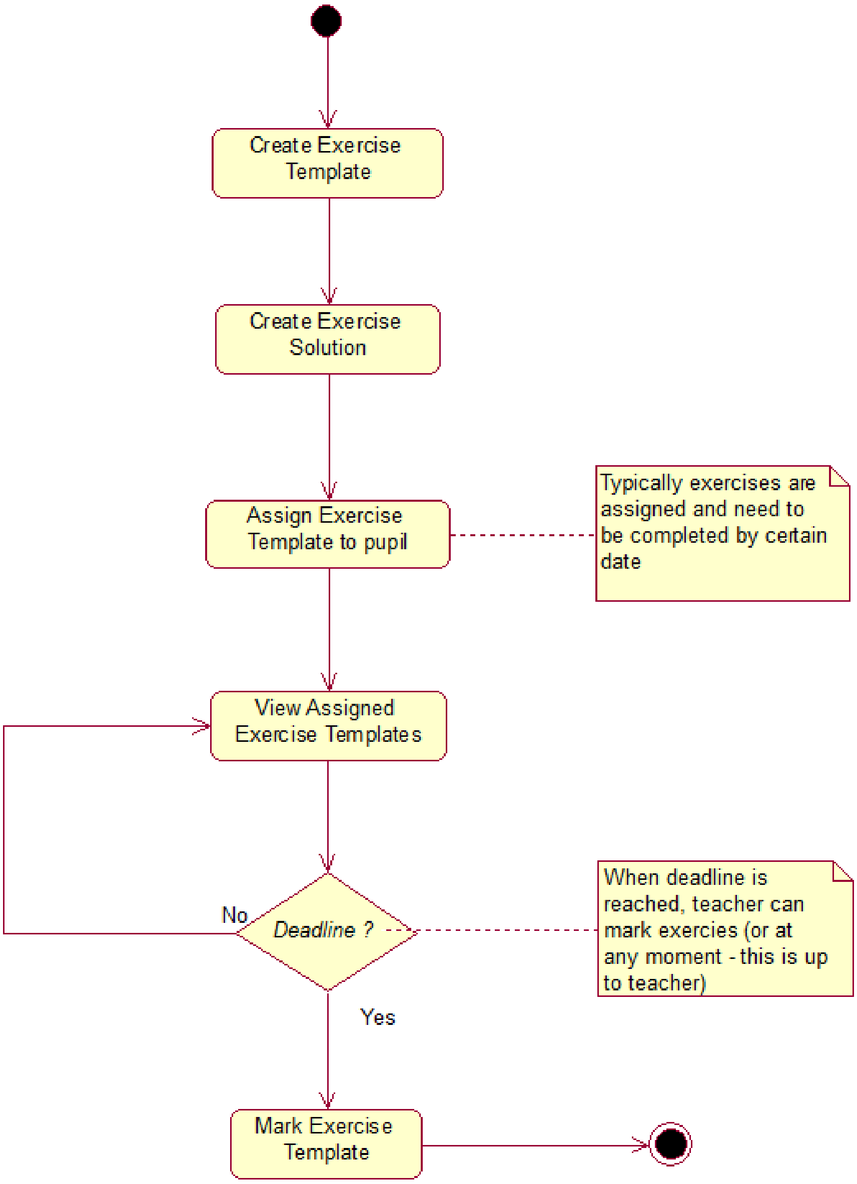

Create Exercise Template
Create Exercise Solution
Assign Exercise Template to pupil
Typically exercises are assigned and need to be completed by certain date
View Assigned Exercise Templates
No
Deadline ?
When deadline is reached, teacher can mark exercies (or at any moment - this is up to teacher)
Yes
Mark Exercise Template

Figure 7 Teacher activity diagram

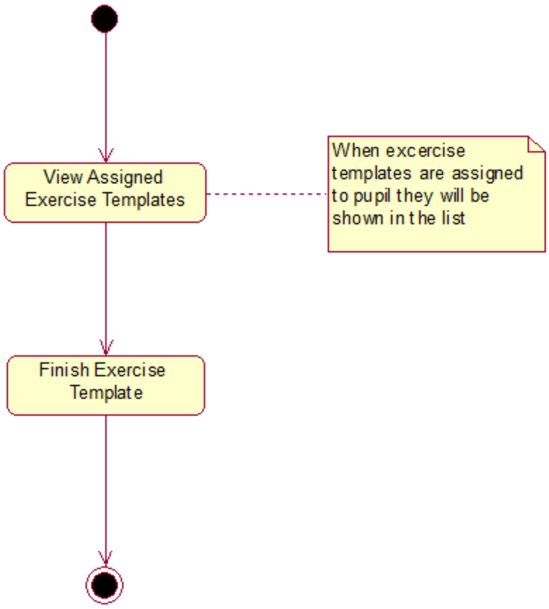

View Assigned Exercise Templates
When excercise templates are assigned to pupil they will be shown in the list
Finish Exercise Template

Figure 8 Pupil activity diagram

1.9.6 Class diagram

The following class diagram describes database structure of the program and relationship between different classes.

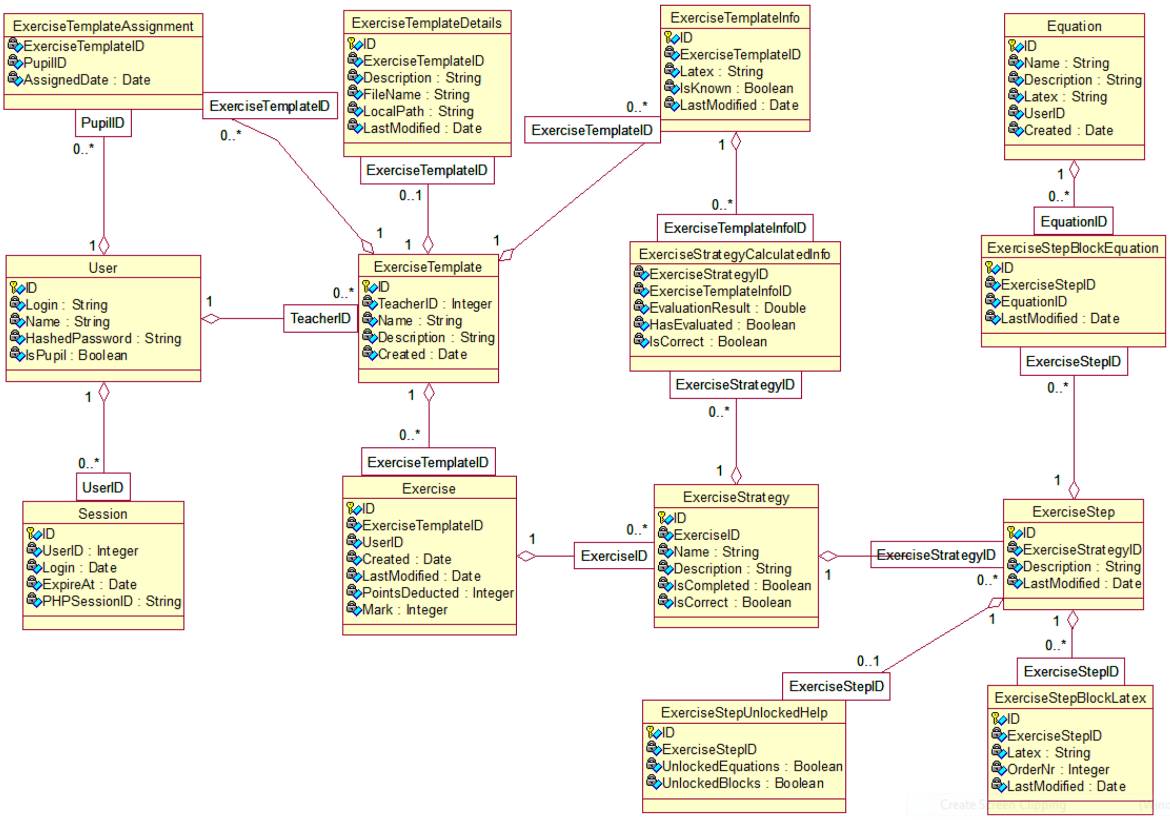


Figure 9 Class diagram of the visual solver

The following classes are defined:

- **User** – an end user of the program, either teacher or pupil
- **Session** – every time user log-in to the system a session object is created with unique ID. After some time of inactivity the session will expire and user has to log-in again. Every request made by the user is also verified if session is valid and if user has required permissions to perform the requested action.
- **ExerciseTemplate** – exercise template created by teacher with name and description
- **ExerciseTemplateAssignment** – assignment of exercise template for specific user
- **ExerciseTemplateInfo** – value that is part of the exercise template description. Every record represents a value that is either known, or should be found in order to complete the exercise.
- **ExerciseTemplateDetails** – additional details for exercise template, added by the teacher. Includes a location of the picture that will be shown for the user.
- **Exercise** – a single exercise which contains multiple strategies that were used to solve associated exercise template.
- **ExerciseStrategy** – a single way of solving exercise template, associated with one exercise.
- **ExerciseStep** – a single step of the exercise strategy.
- **ExerciseStepUnlockedHelp** – a record of pupil using help to unlock part of exercise solution that was made by the teacher.
- **ExerciseStepBlockLatex** – a block with LaTeX expression that will be shown to the user as part of the exercise step.
- **ExerciseStrategyCalculatedInfo** – calculated result of exercise strategy once it is marked as completed, every result is linked with exercise template info that was required to be found.

- **ExerciseStepBlockEquation** – reference to the equation that was used for this step. This is a way for teacher to give a little tip, what kind of equations can be used for solving this step.

- **Equation** – any kind of mathematical expression that can be used for inserting into exercise steps. Typically it is used for storing mathematical equations, but algebraic expression or constants can also be defined.

For a detailed explanation about every field in this class diagram refer to section "Database table structure definitions", 6.1.

1.9.7 Restrictions of the implementation

Due to limited time constraint and vast scale of the topic, the following restrictions will be applied for the solution that will be implemented (scope):

- A solution will be suited for solving the type of exercises where one or more values need to be found and calculated as exact numerical value.

- The limited number of mathematical operations will be supported for user to enter into exercise:

  - Addition, Subtraction, Multiplication, division

  - Trigonometric functions: sin, cos, tan, asin, acos, atan

  - Greek symbols - both capital and regular

- Pictures can be only added by teacher.

- Units of measurement will not be supported.

2 Implemented solution

In this section I will take a look at solution that was implemented, starting with user interface first and then explain general architecture and most important parts of the solution.

The implemented solution and source code is included separately on CD. Refer to CD readme file for installation and configuration instructions. To try solution without installing it, use the following URL http://dmitrikuznetsov.com/ttu/solver/ (Compatible HTML5 browser required such as Chrome, Safari, Opera)

2.1 Program user interface screens

This section is used to describe actual implementation of the program running on tablet device.

Features of this application were tested on iPad Air (iOS7). Following screenshots demonstrate the user interface and functionality of this device.

2.1.1 Program navigational graph

Created program consists of multiple screens. Each screen has a specific purpose. The following image shows navigational options between all of the screens, depending on the user role (pupil or teacher).

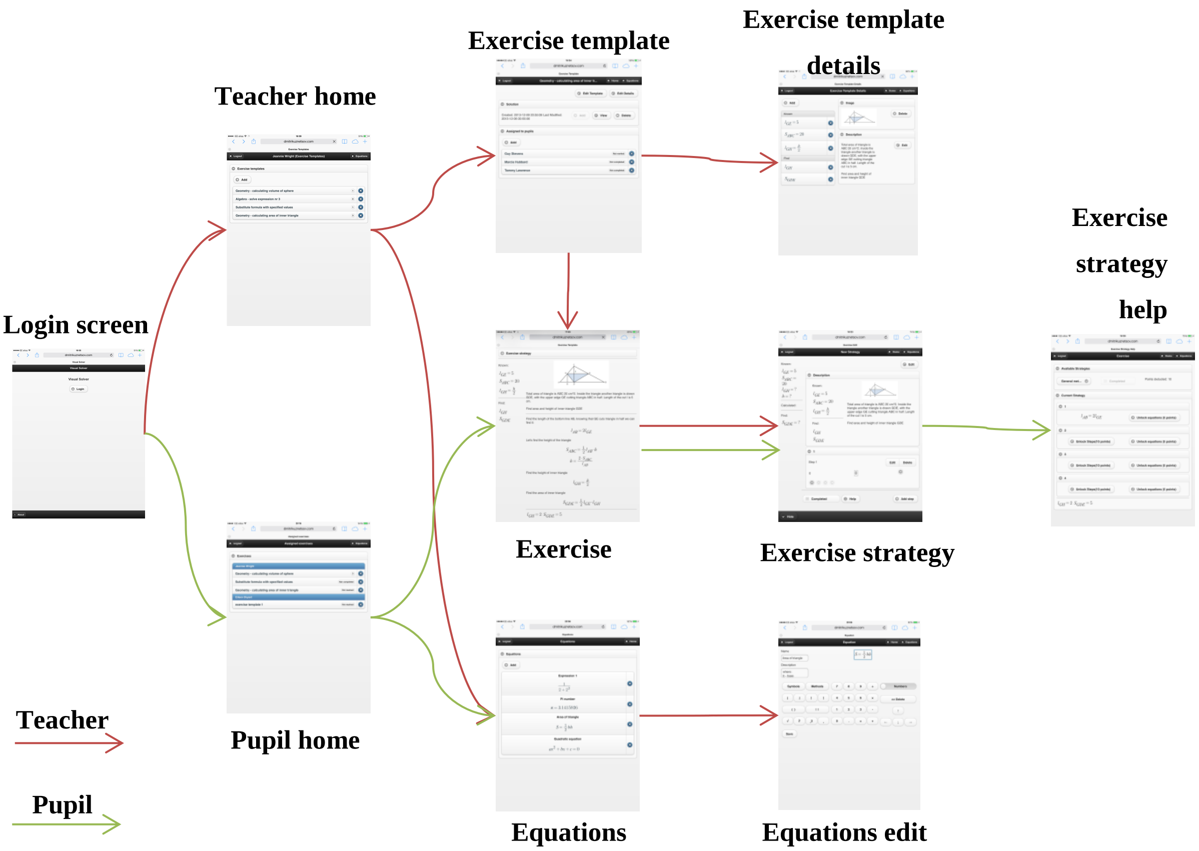


Figure 10 Navigation schema

The breakdown of functionality for each screen is followed in the next sections.

2.1.2 Login screen

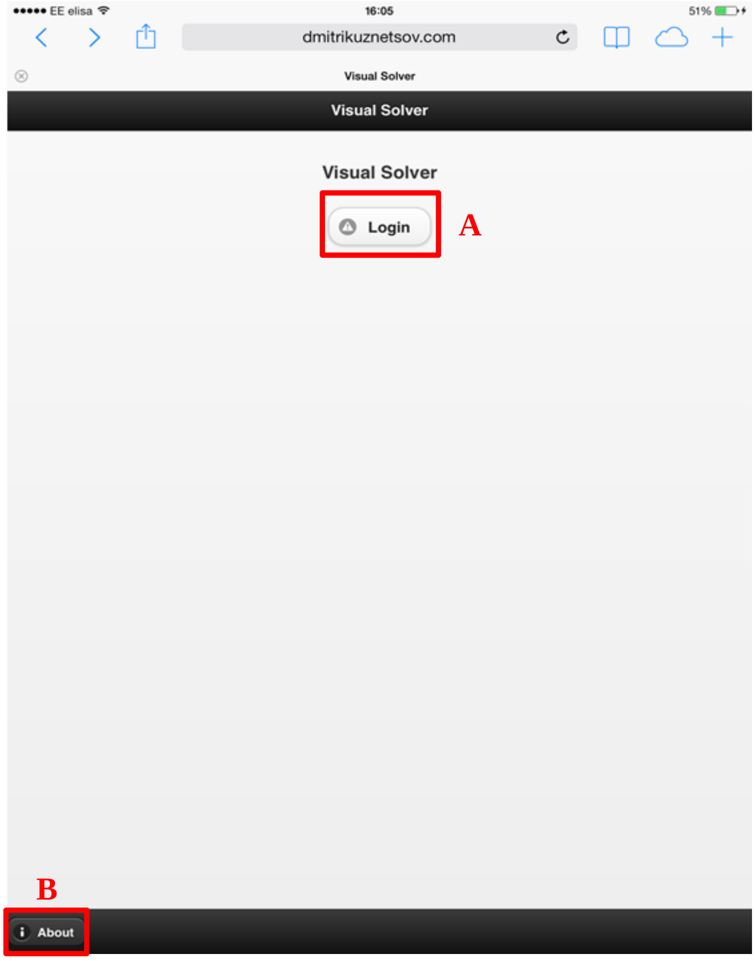
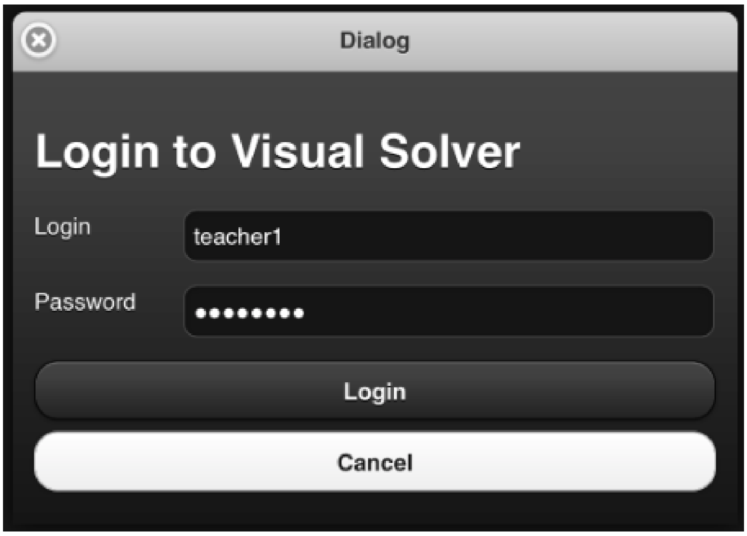


Figure 11 Login screen

Login screen is the first screen user will see when starting the program. This page doesn’t serve any purpose other than asking user for his credentials.

- Login button click (A) – Proceed to home screen. Home screen is determined by the user type. Teacher proceeds to ***Teacher home screen***, and pupils proceed to ***Pupil home screen***. The Login dialog is displayed asking user to specify his credentials:



Dialog

Login to Visual Solver

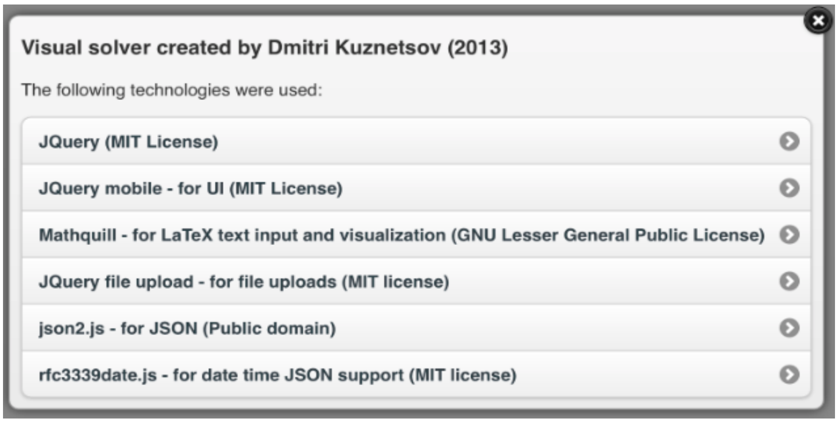
Login: teacher1

Password: ●●●●●●●●●

Figure 12 Login screen dialog

By specifying incorrect credentials an error message will be shown.

- About button click (B) – Displays a popup information dialog about this program, third-party libraries that were used and their licenses:



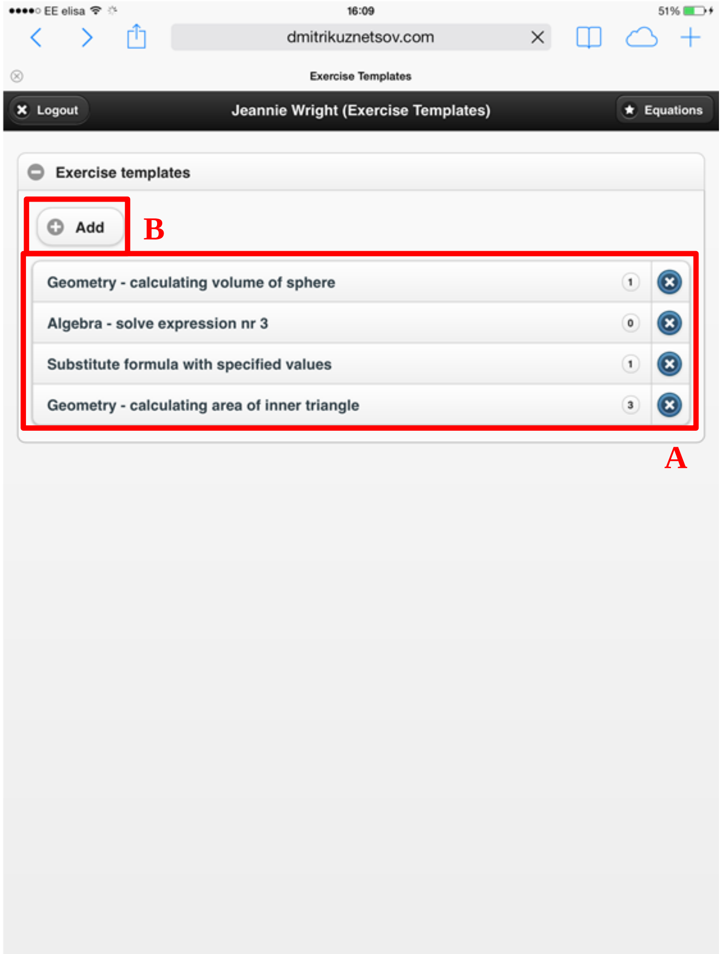
Visual solver created by Dmitri Kuznetsov (2013)

The following technologies were used:

- JQuery (MIT License)
- JQuery mobile - for UI (MIT License)
- Mathquill - for LaTeX text input and visualization (GNU Lesser General Public License)
- JQuery file upload - for file uploads (MIT license)
- json2.js - for JSON (Public domain)
- rfc3339date.js - for date time JSON support (MIT license)

Figure 13 About popup screen

2.1.3 Teacher home screen

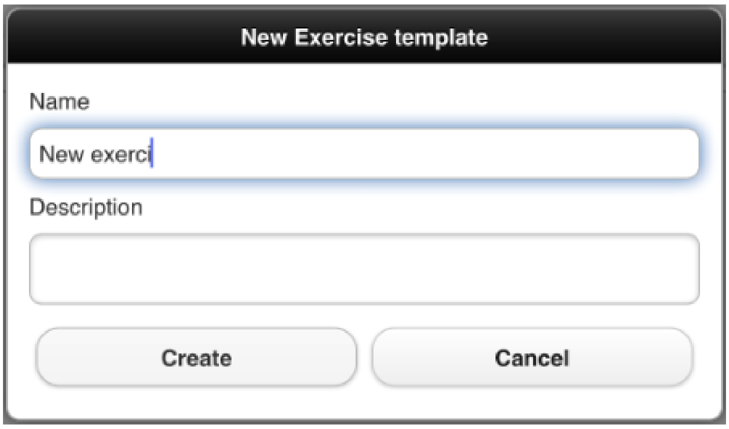


Geometry - calculating volume of sphere 1
Algebra - solve expression nr 3 0
Substitute formula with specified values 1
Geometry - calculating area of inner triangle 3

Figure 14 Teacher home screen

Teacher home screen is used to show information about current exercise templates that teacher has created and their states (A). A list of exercise templates shows: name of the exercise, number of assigned pupils and delete button.

- Exercise template list item click (A) – Clicking on one of the items opens up ***Exercise template screen***.
- Add button click (B) – Opens up a dialog where teacher can specify details for creating new exercise template.



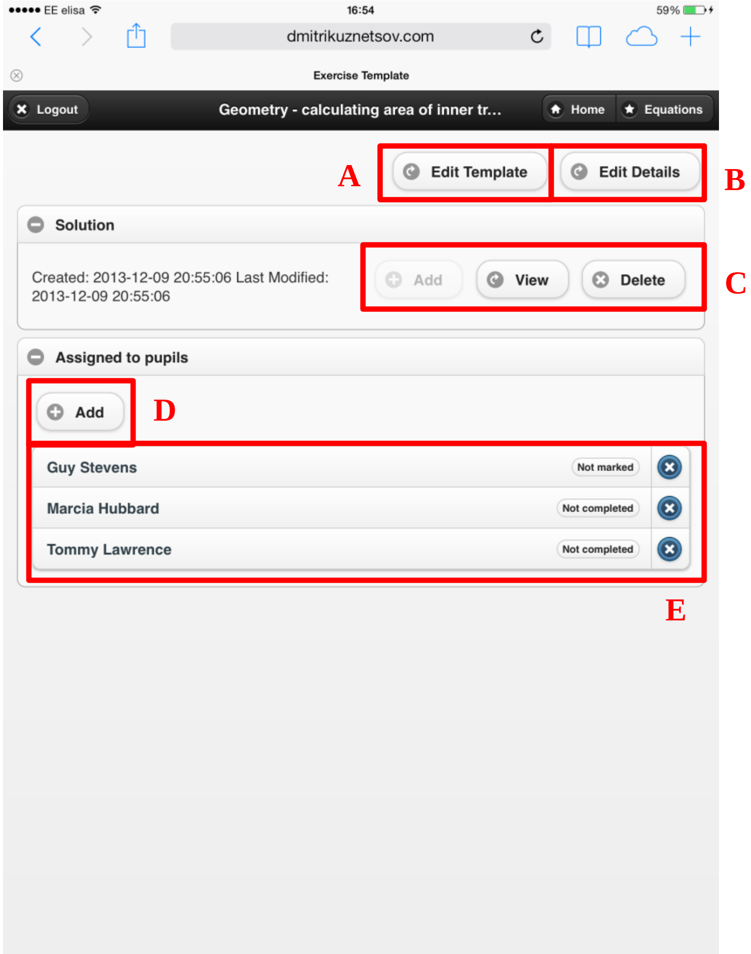
New Exercise template

Name: New exerci

Description:

Figure 15 Add new exercise template screen

2.1.4 Exercise template screen



●●●●● EE elisa 16:54 59%

dmitrikuznetsov.com

Exercise Template

Geometry - calculating area of inner tr...

Solution

Created: 2013-12-09 20:55:06 Last Modified: 2013-12-09 20:55:06

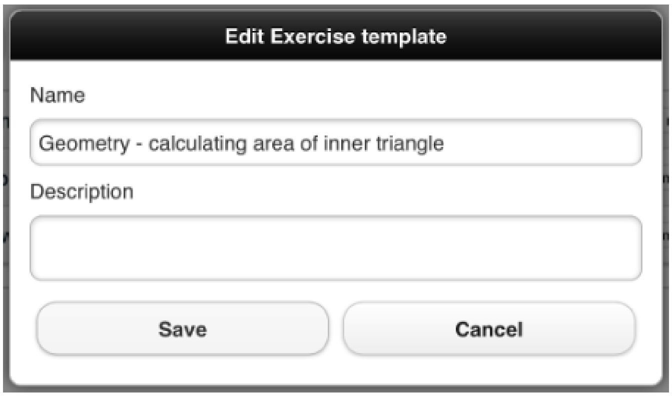
Assigned to pupils

Guy Stevens	Not marked
Marcia Hubbard	Not completed
Tommy Lawrence	Not completed

Figure 16 Exercise template screen

For the teacher this screen provides a good overview over current status of this exercise template. It shows if exercise template itself has all required information and solution before it can be assigned to pupils. The page also shows a list of pupils that have exercise template assigned (E) and status of assignment (if they have completed assignment, received a mark by the teacher or not)

- Edit template button click (A) – Opens up a dialog with basic information about this exercise template:



Name

Geometry - calculating area of inner triangle

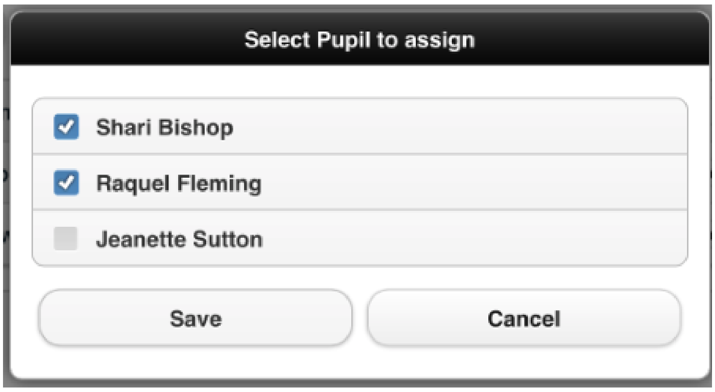
Description

Figure 17 Edit exercise template dialog

- Edit details button click (B) – Opens up ***Exercise template screen*** where teacher can add detailed information about this assignment.

- Edit solution toolbox area (C) – This area is used for managing exercise solution, if it was created for this exercise template. Teacher can create solution for this assignment, so it can be used later as an example for the pupils.

- Add button click (D) – This button is used for assigning new pupils to this exercise template. The dialog will only show pupils that are not currently assigned to this exercise template:



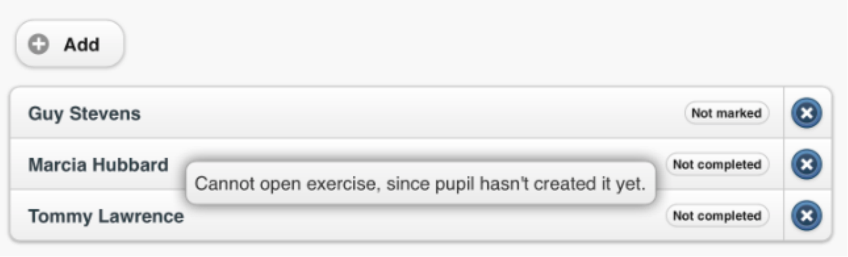
☑ Shari Bishop

☑ Raquel Fleming

□ Jeanette Sutton

Figure 18 Select pupils to assign dialog

- Pupil list item click (A) – when teacher clicks on one of the pupils from the list, he will open up ***Exercise screen*** with exercise that was created by this pupil. If there is no exercise created yet a popup message will appear, explaining this situation:



Guy Stevens — Not marked

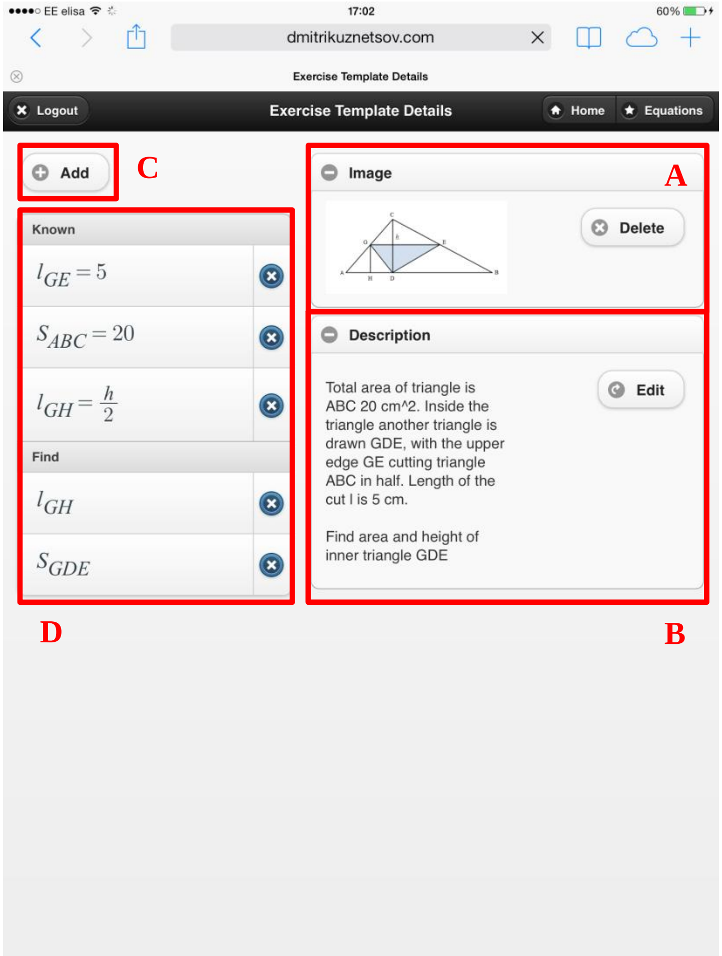
Marcia Hubbard — Not completed

Tommy Lawrence — Not completed

Cannot open exercise, since pupil hasn't created it yet.

Figure 19 Open pupil exercise error popup

2.1.5 Exercise template details screen



Known

$l_{GE} = 5$

$S_{ABC} = 20$

$l_{GH} = \frac{h}{2}$

Find

l_{GH}

S_{GDE}

Total area of triangle is ABC 20 cm^2. Inside the triangle another triangle is drawn GDE, with the upper edge GE cutting triangle ABC in half. Length of the cut l is 5 cm.

Find area and height of inner triangle GDE

Figure 20 Exercise template details screen

This screen is used by the teacher to view or edit exercise template details. Previously in section 1.9.1 I have analyzed the important aspects of tasks and exercises in workbooks.

These elements are present here in almost exactly identical way:

- Picture to visualize the problem (A)
- A short description of the problem (B)
- A list of known values and values that needs to be found (D)
- Clicking Delete button (A) – Will allow teacher to remove existing picture if there is one. After that the picture area will change and allow user to upload another picture:

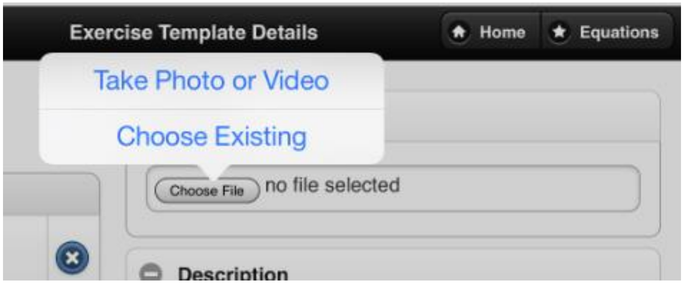
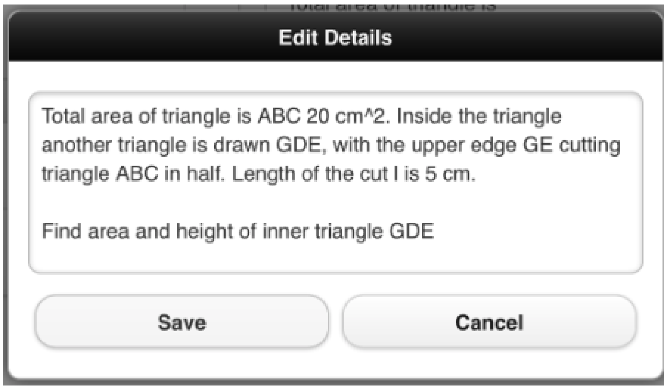


Figure 21 Exercise template details image upload

- Clicking Edit button (B) – Allows teacher to write description of the problem, a dialog will appear:



Total area of triangle is ABC 20 cm^2. Inside the triangle another triangle is drawn GDE, with the upper edge GE cutting triangle ABC in half. Length of the cut l is 5 cm.

Find area and height of inner triangle GDE

Figure 22 Edit exercise template details dialog

- Clicking Add button (C) or list item (D) – Opens up a dialog, where teacher can add/modify values that will be used for solving this exercise. The dialog will include a ***Virtual keyboard*** that can be used on touch devices, since regular keyboard doesn't offer many options for adding mathematical and text items.

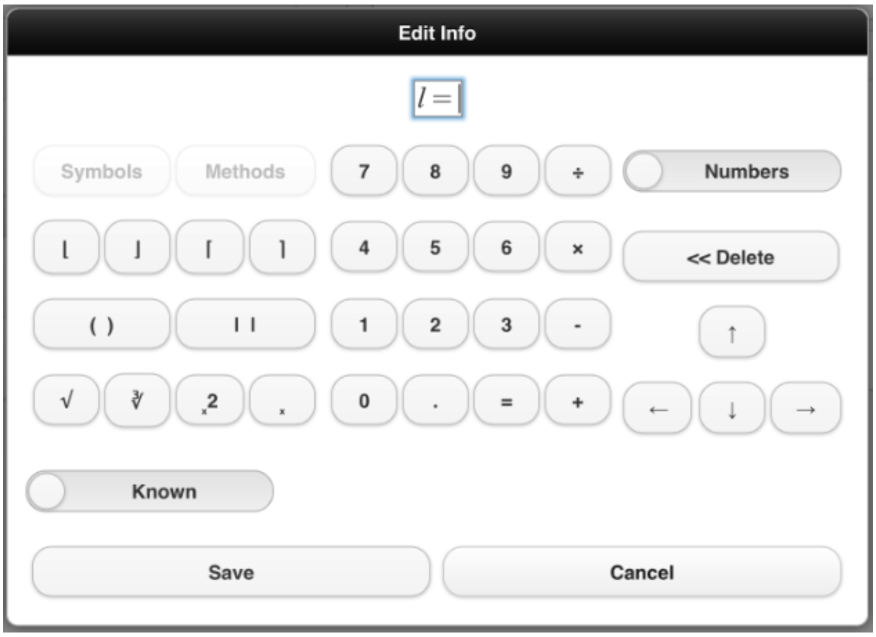
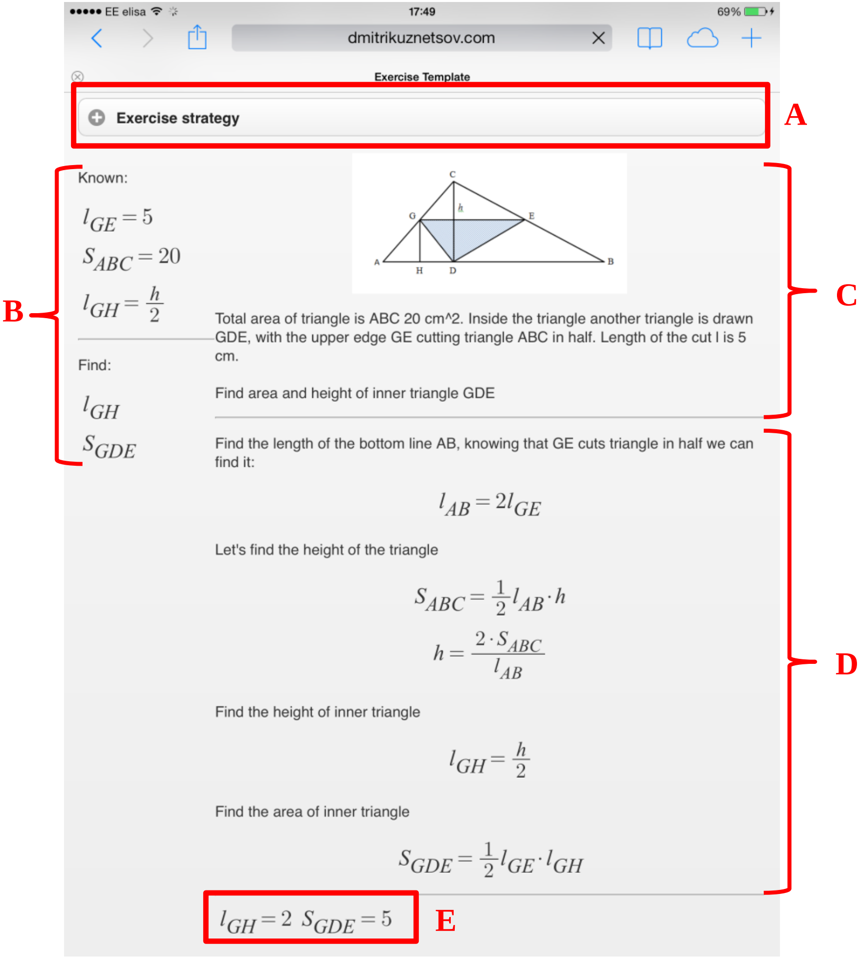


Figure 23 Edit info dialog

To specify if value is known or should be found, click on the toggle button “Known” and Save changes. For breakdown of key elements in virtual keyboard, refer to “Virtual keyboard” in section 2.1.12

2.1.6 Exercise screen



Known:

$l_{GE}=5$

$S_{ABC}=20$

$l_{GH}=\frac{h}{2}$

Find:

l_{GH}

S_{GDE}

Total area of triangle is ABC 20 cm^2. Inside the triangle another triangle is drawn GDE, with the upper edge GE cutting triangle ABC in half. Length of the cut l is 5 cm.

Find area and height of inner triangle GDE

Find the length of the bottom line AB, knowing that GE cuts triangle in half we can find it:

$$l_{AB}=2l_{GE}$$

Let's find the height of the triangle

$$S_{ABC}=\frac{1}{2}l_{AB}\cdot h$$

$$h=\frac{2\cdot S_{ABC}}{l_{AB}}$$

Find the height of inner triangle

$$l_{GH}=\frac{h}{2}$$

Find the area of inner triangle

$$S_{GDE}=\frac{1}{2}l_{GE}\cdot l_{GH}$$

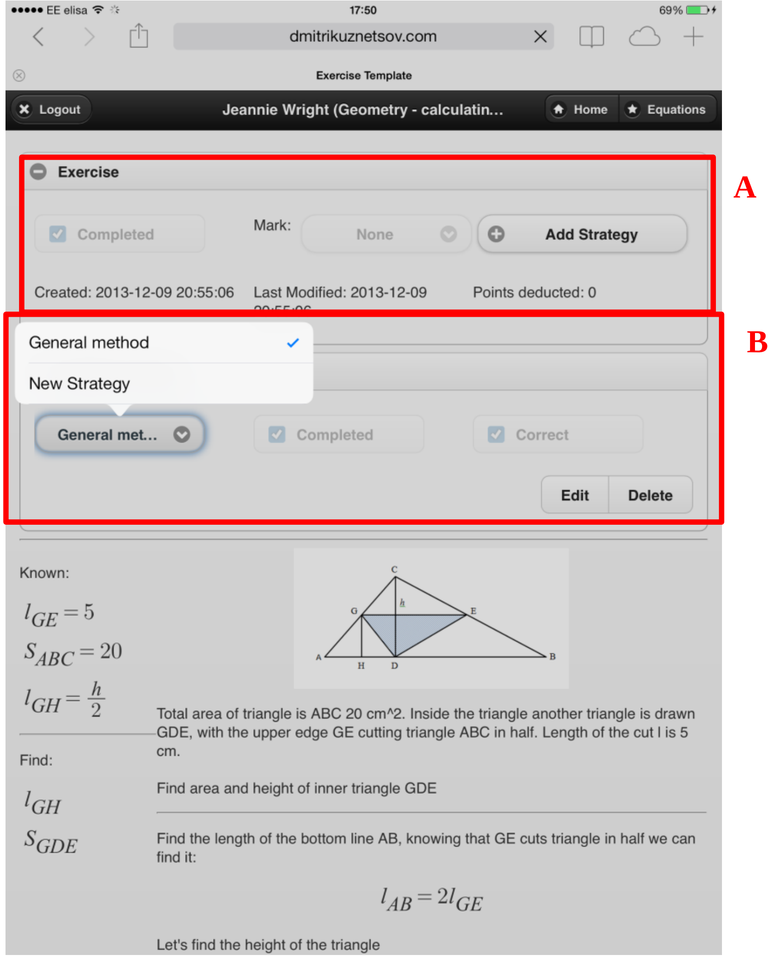
$l_{GH}=2\ S_{GDE}=5$

Figure 24 Exercise screen viewing mode

When ***Exercise screen*** is opened first time it is in viewing mode. By default most of the area is expanded to show exercise strategy steps how assignment was solved, and hide areas that are not important. The areas include:

- A – Exercise strategy selection and other information (Collapsed by default).
- B – Values that were defined by the teacher for this exercise template
- C – Picture that was defined with this exercise template and exercise description
- D – currently selected strategy blocks and steps performed as exercise solution
- E – Final results of values that were defined to be found and their calculated values.

From the teachers perspective, it is important to get a quick look at the exercise that pupil has made, analyze it how it was done. When additional information is needed the upper (A) part can be expanded (Figure 25 Exercise screen edit mode).



Jeannie Wright (Geometry - calculatin...

Exercise

☑ Completed

Mark: None

Add Strategy

Created: 2013-12-09 20:55:06

Last Modified: 2013-12-09 20:55:06

Points deducted: 0

☑ Completed

☑ Correct

Known:

$l_{GE} = 5$

$S_{ABC} = 20$

$l_{GH} = \frac{h}{2}$

Find:

l_{GH}

S_{GDE}

Total area of triangle is ABC 20 cm^2. Inside the triangle another triangle is drawn GDE, with the upper edge GE cutting triangle ABC in half. Length of the cut l is 5 cm.

Find area and height of inner triangle GDE

Find the length of the bottom line AB, knowing that GE cuts triangle in half we can find it:

$$l_{AB} = 2l_{GE}$$

Let's find the height of the triangle

Figure 25 Exercise screen edit mode

In this section, we can see the additional information and current state of this exercise.

Section (A) contains:

- Completed checkbox – Indicates if exercise is completed or not. This mark is set automatically by the server if at least one associated strategy is marked as completed.
- Mark – Mark that was assigned by the teacher for pupil exercise. This cannot be set if teacher is viewing his own exercise solution. For exercises created by pupil, once it receives mark by the teacher, the exercise can no longer be changed.
- Click on Add strategy button – Opens up ***Exercise strategy screen*** for creating new exercise strategy for this exercise.

- Created, Last Modified text – Useful for the teacher to see when exercise was actually created and when last changes were made. Some teachers may have requirements for exercise creation and finishing dates. This way teacher can check if pupils are following established time constraints.

- Points deducted – Number of points that were used by the pupil throughout all exercise strategies when using help tool on ***Exercise strategy help screen***.

Section (B) contains:

- Strategy selector – This selector is used to select what strategy should be shown. By default when exercise is opened the first strategy is selected, user can change this manually.

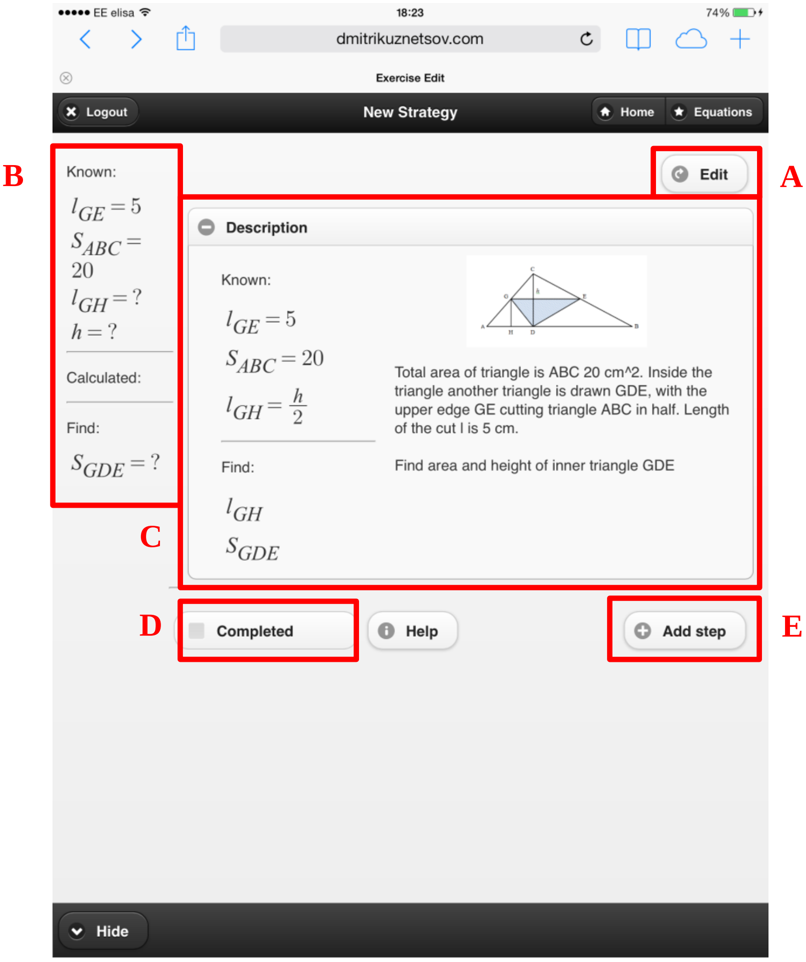
- Completed checkbox – Indicates if user has marked this exercise strategy as completed.

- Correct checkbox – Indicates if all the values that were required to be found, are found and are exactly the same as they are defined in teacher exercise solution. This check is done automatically by server, every time when user marks completed checkbox.

- Clicking Edit button – will open up ***Exercise strategy screen*** where user can make modifications to selected strategy. This is allowed only if exercise was not yet completed or marked by the teacher.

- Clicking Delete button – will allow user to delete selected exercise strategy, this is allowed only if exercise strategy is not marked by the teacher.

2.1.7 Exercise strategy screen



Known:

$l_{GE} = 5$

$S_{ABC} =$ 20

$l_{GH} = ?$

$h = ?$

Calculated:

Find:

$S_{GDE} = ?$

Known:

$l_{GE} = 5$

$S_{ABC} = 20$

$l_{GH} = \frac{h}{2}$

Find:

l_{GH}

S_{GDE}

Total area of triangle is ABC 20 cm^2. Inside the triangle another triangle is drawn GDE, with the upper edge GE cutting triangle ABC in half. Length of the cut l is 5 cm.

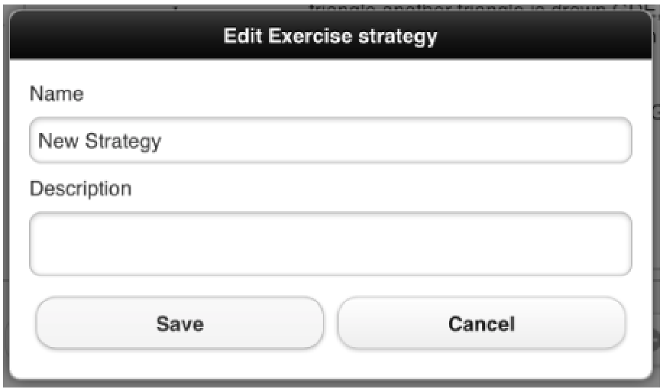
Find area and height of inner triangle GDE

Figure 26 New exercise strategy screen

After creating new exercise strategy, the screen will look something like this (Figure 26 New exercise strategy screen). The user has to start filling the strategy with data in order to solve the exercise.

The following sections are shown:

- Clicking on Edit button (A) – Will allow user to change name and description of this strategy:



Edit Exercise strategy

Name

New Strategy

Description

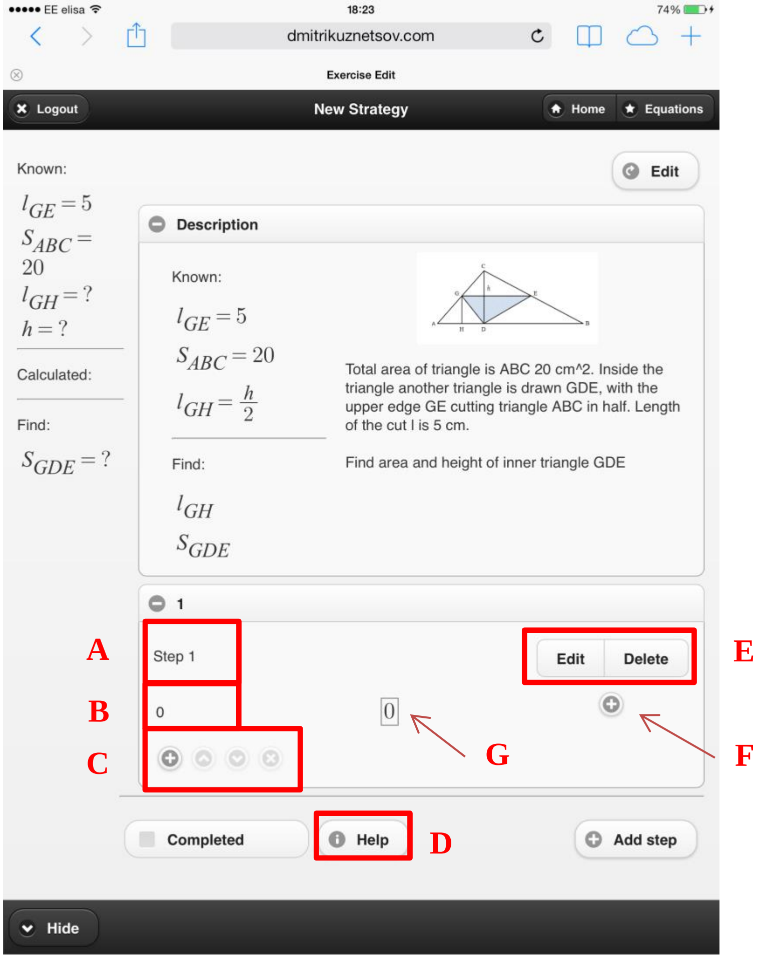
Figure 27 Exercise strategy edit dialog

- Variables panel (B) – Is used for showing calculated values. The panel shows values that are unknown with “?”. Once the user starts to fill strategy steps, the values list will be automatically updated to show what is known at the moment. This is useful for the user to see how values change in real-time, without interacting with the panel directly.

- Description panel (C) – Is used to show exercise assignment details. This was specified by the teacher for exercise template in ***Exercise template details screen***.

- Clicking on Completed button (D) – this will mark exercise as completed. The calculated values will be sent to server, where they will be checked for values in existing solution (for pupil), for teacher values are always assumed to be correct.

- Clicking on Add step button (E) – will create a new step (Figure 28 Exercise strategy screen with exercise step).



Known:

$l_{GE} = 5$

$S_{ABC} = 20$

$l_{GH} = ?$

$h = ?$

Calculated:

Find:

$S_{GDE} = ?$

Known:

$l_{GE} = 5$

$S_{ABC} = 20$

$l_{GH} = \frac{h}{2}$

Find:

l_{GH}

S_{GDE}

Total area of triangle is ABC 20 cm^2. Inside the triangle another triangle is drawn GDE, with the upper edge GE cutting triangle ABC in half. Length of the cut l is 5 cm.

Find area and height of inner triangle GDE

$\boxed{0}$

Figure 28 Exercise strategy screen with exercise step

When new step is added, a block will appear on the bottom of the page. Let's break it apart:

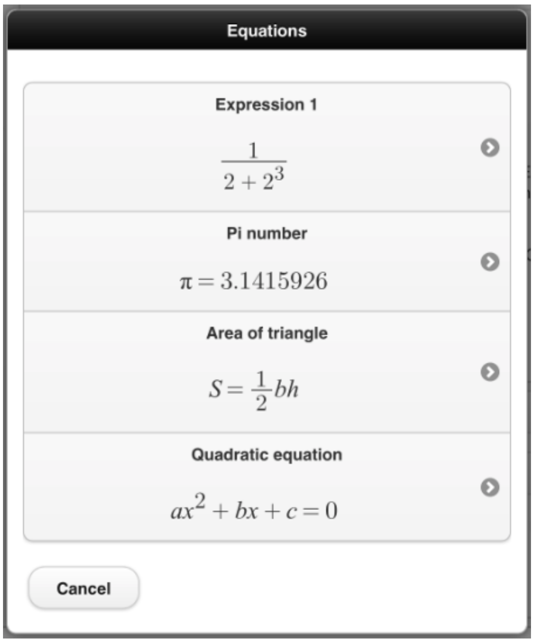
- Step description (A) – Can be used to show what is the main purpose of this step (what kind of operations are made here). Can be changed by clicking Edit button (D).
- Evaluation of a block (B) – Since every step can contain multiple blocks, every block is evaluated in real-time when user changes any block. The value of the evaluation is written on the left side. In this case block (G) evaluated to 0, and the value is written to (B).
- Block manipulation buttons (C) (from left to right):
    - Add – Add new block to this step.
    - Move up – Moves selected block to the top.
    - Move down – Moves selected block to the bottom.

    - Delete – Deletes selected block.

- Clicking Help button (D) – Allows pupil to request help for this exercise, this opens up ***Exercise strategy help screen***. Visible only to pupil.

- Step control buttons (E) – Allows user to edit selected step description or delete this step.

- Add equation button (F) – Opens up an equation selection dialog, where user can select equation from the list to be added into this step.



Equations

Expression 1

$$\frac{1}{2+2^3}$$

Pi number

$$\pi = 3.1415926$$

Area of triangle

$$S = \frac{1}{2}bh$$

Quadratic equation

$$ax^2 + bx + c = 0$$

Figure 29 Equation selection dialog

After any item is selected from the list, it will be added to this step as it is displayed in the list. After it is added, user can manually change it, and substitute variables with the ones that are used in current exercise:

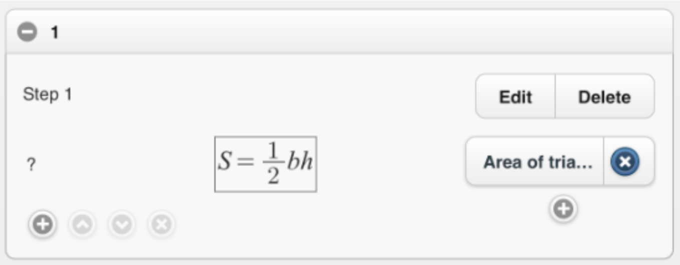
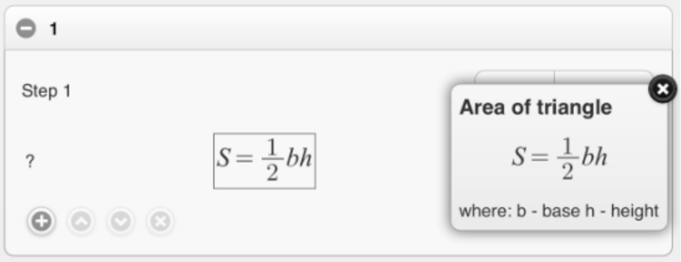


Figure 30 Exercise strategy step with added equation

User can click on any of the items in equation list (on the right) and get a brief description of the equation details:



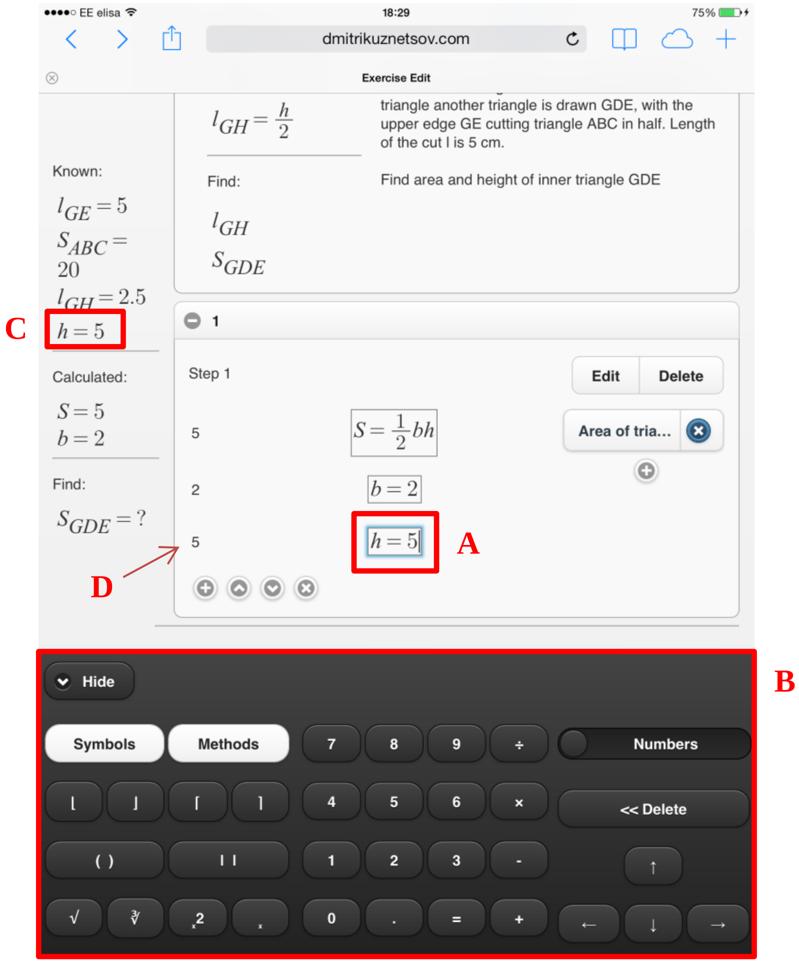
Area of triangle

$$S = \frac{1}{2}bh$$

where: b - base h - height

Figure 31 Exercise strategy step with equation info

- Single editable block (G) – when block has a black border around it, this indicates that block is in editable mode, and can be changed by the user.



triangle another triangle is drawn GDE, with the upper edge GE cutting triangle ABC in half. Length of the cut l is 5 cm.

Find: Find area and height of inner triangle GDE

$l_{GH} = \frac{h}{2}$

Find: l_{GH} S_{GDE}

Known: $l_{GE} = 5$ $S_{ABC} = 20$ $l_{GH} = 2.5$ $h = 5$

Calculated: $S = 5$ $b = 2$

Find: $S_{GDE} = ?$

Step 1

5 $S = \frac{1}{2}bh$

2 $b = 2$

5 $h = 5$

Figure 32 Exercise strategy in editable mode

Let's take a look at editable mode more closely (Figure 32 Exercise strategy in editable mode). When the user clicks on a block (A), the ***Virtual keyboard*** will popup (B). ***Virtual keyboard*** allows user to enter values into text box, on touch screen devices where there is no physical keyboard. ***Virtual keyboard*** includes all the functionality that is currently supported by the program mathematical solver, so any method can be evaluated as long as it is inserted correctly.

As user is typing text, the program will automatically save changes to the server, so there is no need to have save button. Also this will cause all related expressions to be re-calculated. In the example above the block has evaluated to 5 (D), and since it is equal to *h* it was also evaluated to 5 and shown on the variables panel (C). Any other expression or equation that depends on this value will be evaluated as well.

2.1.8 Exercise strategy help screen

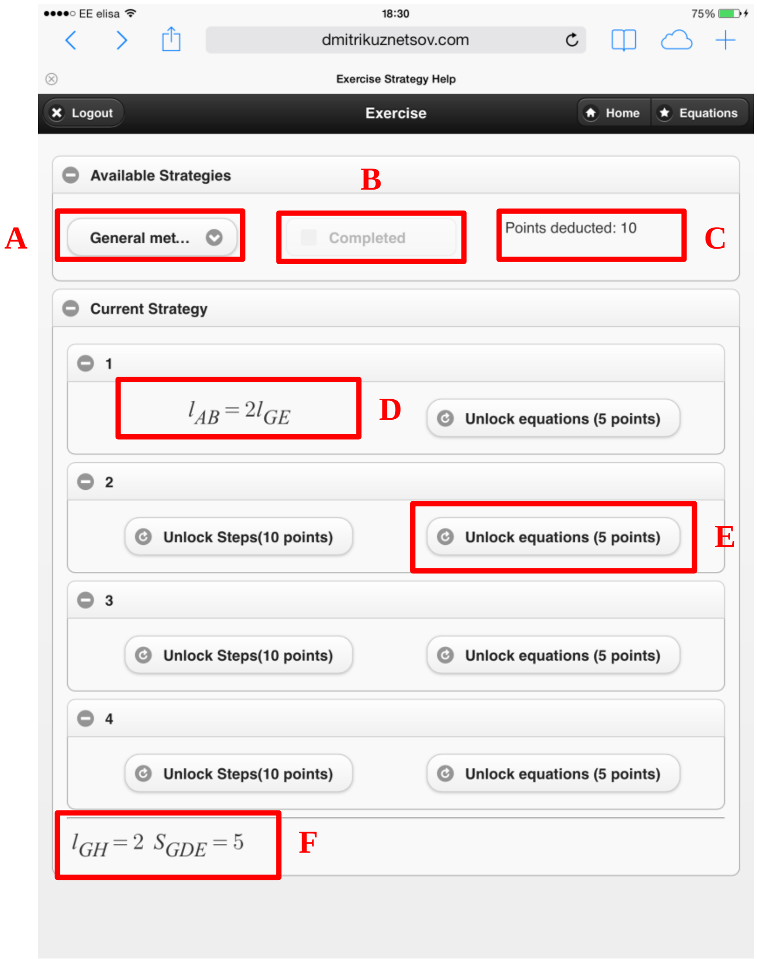


Figure 33 Exercise strategy help screen

The help screen allows pupil to request help when he is stuck trying to solve the exercise. One of the ideas behind this approach is that in order to continue, pupil can unlock only those portions that he has trouble with, so in the end he will try to finish assignment himself instead of abandoning all the work. For any unlocked block though, he will accumulate points (C). These points can be used later by teacher to decide what kind of mark this exercise can be assigned.

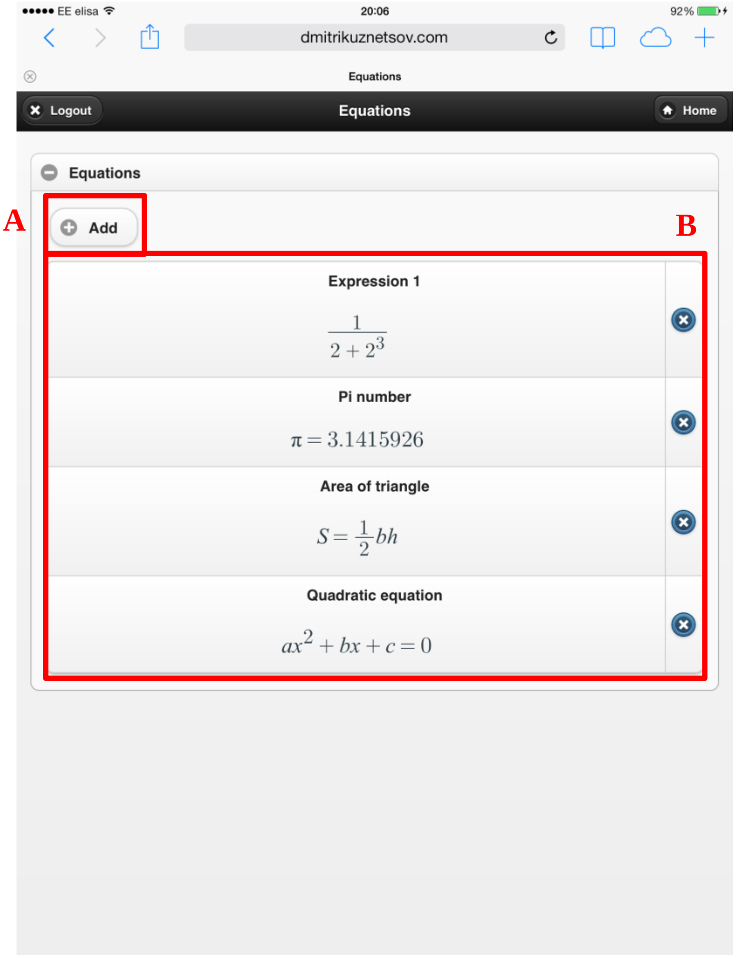
- Strategy selector (A) – can be used to select strategy solution from the list of strategies that were created for this exercise by teacher.
- Completed checkbox (B) – indicates if teacher has marked this strategy as completed.
- Number of points that were deducted so far (C) – Every time unlock is made, this number will increase across all strategies that were created for this exercise by pupil.

- Unlocked block (D) – Once block is unlocked, it will remain unlocked so the pupil can view it again any time when it is required.

- Locked block (E) – Locked block that can be unlocked by pupil, for a few points.

- Results of this strategy that were obtained by teacher (F) for this selected strategy solution.

2.1.9 Equations screen



Expression 1

$$\frac{1}{2+2^3}$$

Pi number

$$\pi = 3.1415926$$

Area of triangle

$$S = \frac{1}{2}bh$$

Quadratic equation

$$ax^2 + bx + c = 0$$

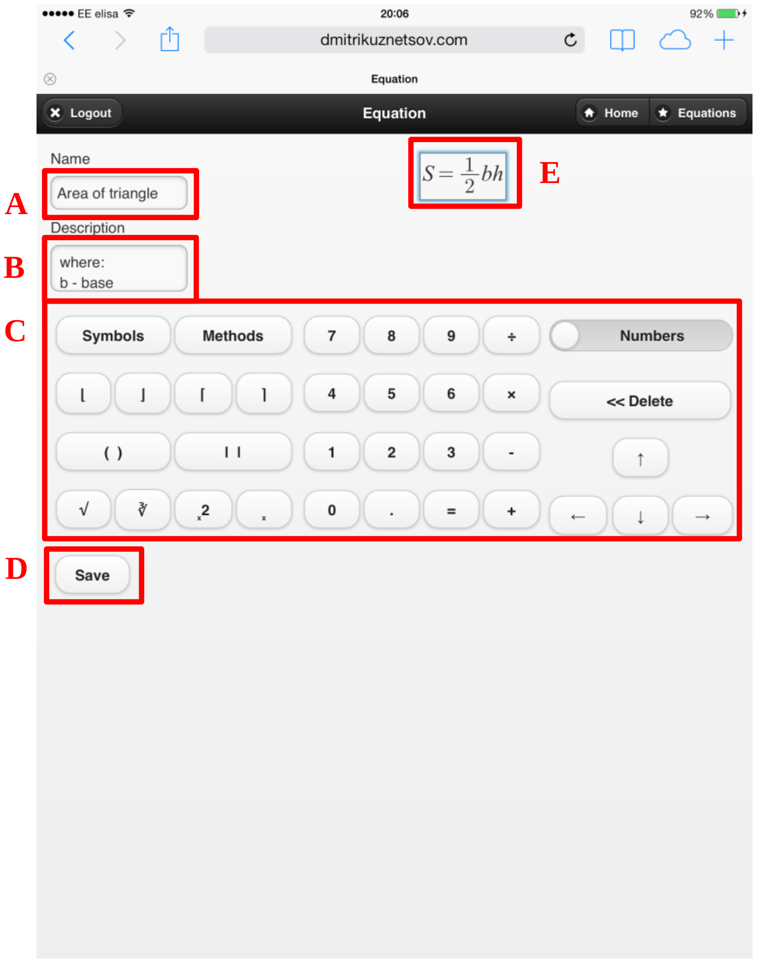
Figure 34 Equations screen

Primary purpose of this screen is to hold equations that can be later added to exercises. The page can also be used to add algebraic expressions or even constants. There is no restriction on the type of data that will be stored here, as long as it can be added to exercise calculations later.

- Clicking on Add button (A) – Will open ***Equation edit screen*** where teacher can add new equation to the list.

- Clicking on list element (B) – will open existing item in ***Equation edit screen***, so it can be modified.

2.1.10 Equation edit screen



Name

Area of triangle

Description

where:
b - base

$S=\frac{1}{2}bh$

Figure 35 Equation edit screen

This screen is used to modify or add new equations to the system. The following sections are shown:

- Name of the equation (A)

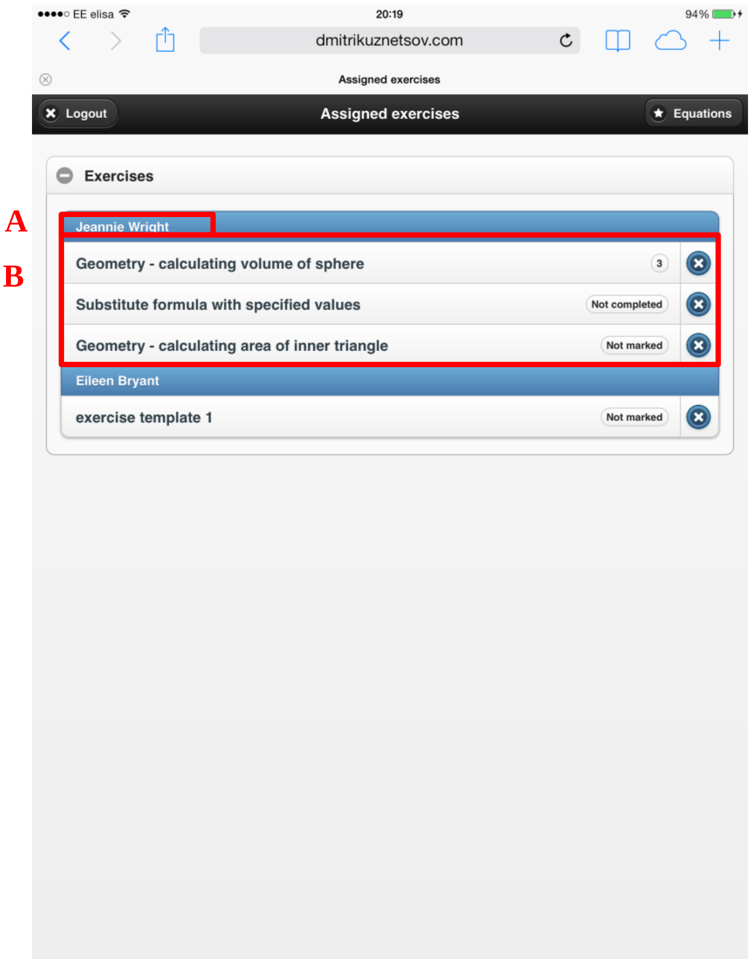
- Description of the equation (B).

- Virtual keyboard (C)

- Clicking on save button (D) - will save changes that were made.

- Text input area (E) where equation can be written.

2.1.11 Pupil home screen



Exercises

Jeannie Wright

Geometry - calculating volume of sphere — 3

Substitute formula with specified values — Not completed

Geometry - calculating area of inner triangle — Not marked

Eileen Bryant

exercise template 1 — Not marked

Figure 36 Pupil home screen

After log-in pupil will see the following screen. Pupil home screen shows a list of current exercise assignments and their status (B). This gives pupil a rough idea what kind of exercises need to be done, or if they have been done.

- Every teacher (A) assignments are separated, so the pupil can find related exercises quicker.

- Assignment list shows the following:
    - Exercise template name

- Current status – if exercise was completed, and what mark it has received.

- Delete button – deletes existing exercise, if it hasn’t yet been marked by teacher.

2.1.12 Virtual keyboard

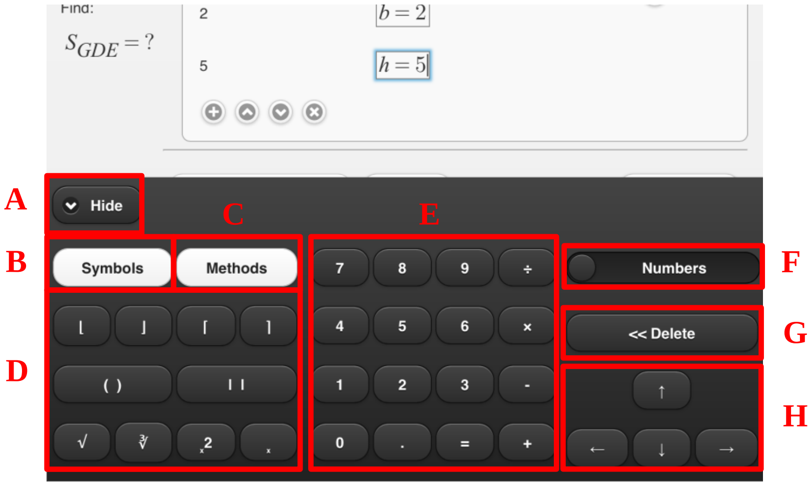


Figure 37 Virtual keyboard number mode

Every time input needs to be inserted as math, the virtual keyboard will be used. Let’s break down parts of the keyboard that are shown on the picture above:

A. Hide virtual keyboard button - When this button is pressed, the keyboard will disappear. In order to open keyboard again, click on the math block again that you would like to edit.

B. Symbols popup menu. Press it to open symbol selection popup:

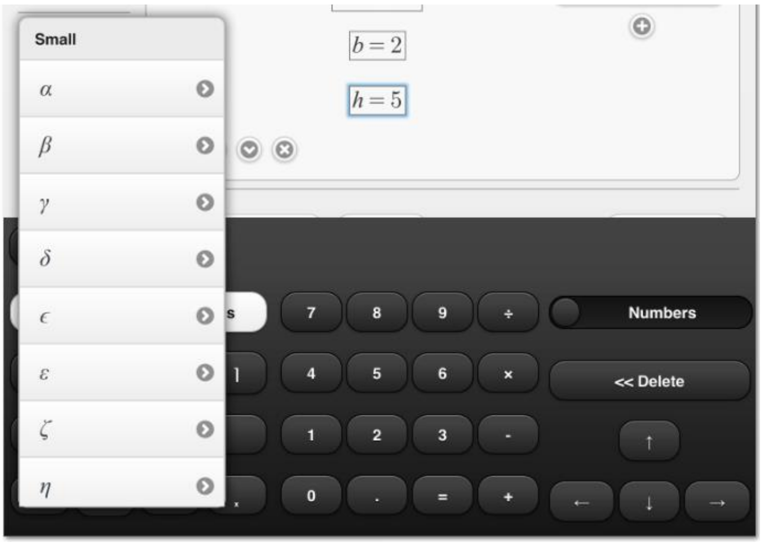


Figure 38 Virtual keyboard symbols popup

Most commonly used symbols are included in the list, scroll through the list to view remaining symbols.

C. Methods popup menu. Press it to open method selection popup:

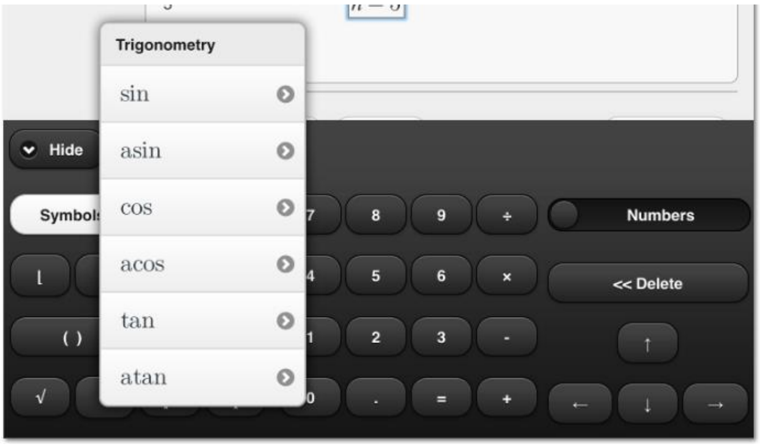


Figure 39 Virtual keyboard method popup

Currently most commonly used trigonometry methods are available.

D. List of operators and brackets (from left to right):

a. Floor

b. Cap

c. Parentheses

d. Absolute value

e. Square root

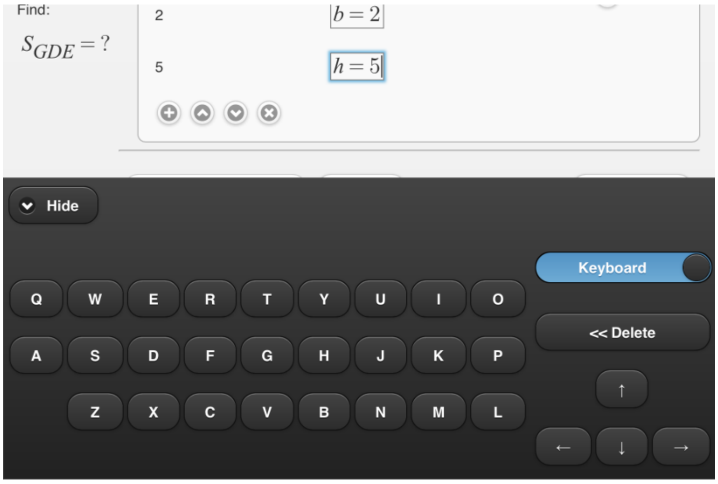
f. Cube root

g. Power of two

h. Subscript

E. Numeric keypad (numbers and common operators)

F. Numbers or Keyboard toggle button. This can change the layout to allow characters to be inserted with virtual keyboard:



Find:

$S_{GDE} = ?$

2 $b = 2$

5 $h = 5$

Figure 40 Virtual keyboard keys mode

G. Delete button. Removes any value at the current cursor position.

H. Navigational keys, allows user to move cursor in specified direction.

2.2 Design and software architecture

2.2.1 General high level architecture

This section explains general high level architecture of this program and why it was chosen.

This program was created using Client-Server model. In this model the resources (exercises, assignments or other data) are stored on the server and shared between users (clients) that communicate with the server.

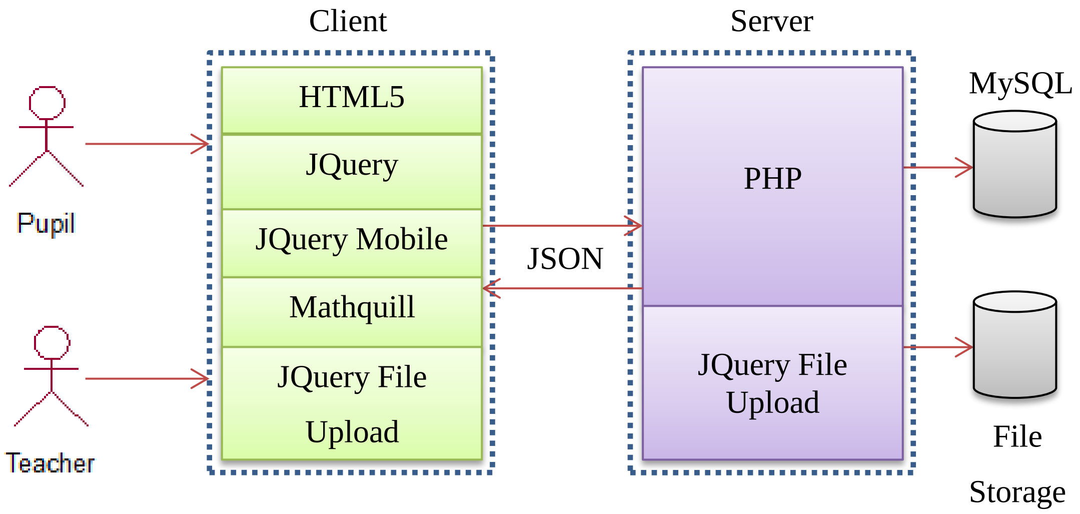


Figure 41 General high level architecture

2.2.1.1 Server side architecture

For the server side, the following technologies were selected

- PHP – "Hypertext Preprocessor" is a popular general-purpose scripting language. It is used for server side code [26]. The decision to use PHP is based on the fact that it is popular with many hosting providers and free for everyone to install and use. PHP is also available on many popular platforms (Windows, Mac and Linux). It is fairly easy to learn, has large and active community, well written documentation and examples.

- MySQL – For collaboration purposes, server database storage is required. MySQL is the most popular open source database in the world [27]. It is free as well, so it can be installed and used alongside with PHP. For using MySQL via PHP a MySQLi extension is used (http://www.php.net/mysqli).

One of the main advantages of using both PHP and MySQL together, is that they can be installed from a single package, called WAMP (for windows - http://www.wampserver.com/en/), LAMP (Linux - http://lamphowto.com), MAMP (Mac OS - http://www.mamp.info). This means that installation and set-up of the server can take just a few minutes and can be done with a single straightforward installation process. It is very convenient to manage later as well, since packages include a special server managing tool for this purpose.

2.2.1.2 Client side architecture

For the client side, the technologies chosen depend on the requirements that were initially set (for the type of device to be used in section 1.7 and analysis in section 1.9.1)

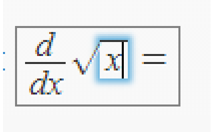
- Web, HTML5, JavaScript – The software is written for the web using HTML5 standard [13]. Most of the HTML5 features are already widely supported on all modern browsers. This allows application to work on any device that has compatible browser installed (such as tablets, phones, laptops). HTML5 also includes many new features and technologies that allow web applications to have rich and flexible content. This technology will make it possible to implement many new features that were exclusive to desktop computers just a few years ago. A possible alternative would be Adobe Flash [28], however it requires additional plugin to be installed. Flash is also commercial software and costs a certain fee for development, not all modern devices have support for it.

- JQuery – Fast, small and feature-rich JavaScript library [29]. JQuery is one of the most powerful and popular libraries for JavaScript available today. It supports many features that are used in this project: “Events”, “Ajax requests”, “HTML inject”, “Selectors” and others. This is essential for building program faster. Some other libraries require JQuery to be included so it has to be present anyway.

- JQuery Mobile – A unified HTML5 based user interface, designed for popular mobile platforms [30]. Since development will be primary focused on implementing solution for mobile devices that have touch interface (tablets), it is important that interface will be designed accordingly and this is where JQuery mobile fits perfectly. There are alternatives, some of them are commercial software, some are free, but for the number of features and no cost, there is very little competition. JQuery Mobile requires JQuery to be present.

- Mathquill – A JavaScript library that allows user to type into text box and turn it into visual representation of math [31]. Mathquill works by converting LaTeX text. LaTeX is a document preparation system [32].

For example, the following text box on the web page was marked to be managed by Mathquill. As user is typing text, a math is created automatically (storing it internally as LaTeX string):



$\frac{d}{dx}\sqrt{x} =$

Figure 42 Typing text into mathquill enabled text box

```
\frac{d}{dx}\sqrt{x}=
```

Figure 43 LaTeX text that was generated by Mathquill

This process can be reversed and existing LaTeX text can be assigned to another text box.

Note: some modifications to Mathquill library were made. Mathquill is not yet a release version, so many features are missing. Mathquill was not designed to work with ***virtual keyboard***, so it is not possible to manipulate LaTeX text without making some changes to the library manually. Features that were added:

1 Ability to show current position of cursor

2 Ability to move cursor in specific direction (left, right, top, bottom)

3 Ability to delete last text manually.

2.2.1.3 Communication architecture

For communication between server and client, the following technologies are used

- JQuery File Upload – Is used for storing files on the server (such as pictures) it is File Upload widget with multiple file selection, drag and drop support, progress bar, validation and preview images, audio and video for jQuery [33]. For this program only single file upload feature is used. The plugin includes both server and client parts and is free to use as well.

- JSON – “JavaScript Object Notation”. Since information between server and client needs to be exchanged, some sort of standard need to be agreed on by both sides. JSON is a good choice for this. Since JSON has native support starting from ECMAScript Edition 5 [34] and PHP v 5.2 [35] they both can communicate natively.

This makes it ideal choice for a way to exchange information without any additional plugins or tweaks. Another important feature when using JSON is that information is stored in “plain text”, this means that information encoded in JSON can be read by anyone, to debug application problems quicker.

```
19:53:24 (engine) [RESPONSE (User.GetExercise)]->array(3) {
 ["result"]=>
 string(2) "OK"
 ["data"]=>
 array(9) {
  ["UserName"]=>
  string(11) "Guy Stevens"
  ["UserID"]=>
  string(1) "3"
  ["UserIsPupil"]=>
  string(1) "1"
  ["ExerciseTemplateID"]=>
  string(1) "7"
  ["IsCompleted"]=>
  string(1) "1"
  ["PointsDeducted"]=>
  string(2) "10"
  ["Mark"]=>
  string(2) "-1"
  ["Created"]=>
  string(19) "2013-12-09 23:20:55"
  ["LastModified"]=>
  string(19) "2013-12-09 23:20:55"
 }
 ["total"]=>
 int(1)
}
```

Table 11 Example of server response message to client written to internal log

Even though JSON support is included natively, during development few issues were found. Additionally json2.js (http://javascript.crockford.com/) and rfc3339date.js (by Paul GALLAGHER http://tardate.com) had to be included in the project. Since JSON and PHP doesn’t support Date Time values. The transferred dates were missing some values or parsed incorrectly. Included libraries fix this issue.

2.2.2 Security considerations

The following security considerations were made:

- Log-in is required for any user (teacher or pupil)
- Password is hidden when typing

- If log-in was successful the session is created for this user, with expiry date. If user is inactive for some time the session expires and user has to log-in again.

- Actions that can be performed only by teacher cannot be performed by the pupil (for example pupil cannot assign mark for exercise by himself)

- Other users cannot modify work of another user (for example one pupil cannot make changes to another pupil exercise)

- File uploads are allowed only for image files (uploads are done by JQuery Mobile and files are checked automatically before upload starts). To start any upload user must have a valid session.

- For many actions, checks are done on the server side. For example, verification that exercise was completed correctly is performed on server side, this is done to be sure that the results were not manipulated on client side.

- Server should use SSL if available; otherwise password information could be visible on the network as it is transferred in plain text. For the demo application it is working on normal HTTP server not HTTPS.

2.2.3 Mathematical solver

One of the most important parts of this program is a solver for mathematical equations and algebraic expressions. The solver is made on client side using JavaScript. The following section describes a method how it is implemented.

2.2.3.1 Using JavaScript as a tool for evaluation

One of the biggest advantages of JavaScript is a built-in evaluation method. Simply writing an expression and calling ***eval()*** will perform evaluation by JavaScript and return a result, this can be used for writing simple calculations like that:

JavaScript code	**Result**
eval("2+2")	4
eval("2+(2*4)")	10

eval("3+2^2-4")	-5
eval("Math.abs(-3)")	3
eval("Math.pow(2, (2))")	4
eval("(2 + 2)/(1 + 1)")	2

Table 12 Example of JavaScript evaluation results

This means that there is already available functionality for calculating simple expressions.

2.2.3.2 Creating LaTeX to JavaScript expression conversion

Every exercise strategy consists of many steps and blocks, each of them contains LaTeX expression. In order to calculate result of one block we need to parse LaTeX expression and convert it to JavaScript expression. In the project code this is done by LatexSolver.js class.

Since we cannot support all possible combinations of LaTeX expressions, the best idea is to start from JavaScript and analyze what kinds of methods are supported by JavaScript.

Here is a list of methods that are supported by JavaScript Math.* library (http://www.w3schools.com/jsref/jsref_obj_math.asp):

Math method	**Description**	**Is Supported by solution?**
Math.abs(x)	Returns the absolute value of x	YES
Math.acos(x)	Returns the arccosine of x, in radians	YES
Math.asin(x)	Returns the arcsine of x, in radians	YES
Math.atan(x)	Returns the arctangent of x as a numeric value between -PI/2 and PI/2 radians	YES
Math.atan2(x)	Returns the arctangent of the quotient of its arguments	NO
Math.ceil(x)	Returns x, rounded upwards to the nearest integer	YES
Math.cos(x)	Returns the cosine of x	YES
Math.exp(x)	Returns the value of E^x	NO
Math.floor(x)	Returns x, rounded downwards to the nearest	YES

	integer	
Math.log(x)	Returns the natural logarithm (base E) of x	NO
Math.max(x,y,z…n)	Returns the number with the highest value	NO
Math.min(x,y,z…n)	Returns the number with the lowest value	NO
Math.pow(x,n)	Returns the value of x to the power of y	YES
Math.random()	Returns a random number between 0 and 1	NO
Math.round(x)	Rounds x to the nearest integer	NO
Math.sin(x)	Returns the sine of x (x is in radians)	YES
Math.sqrt(x)	Returns the square root of x	YES
Math.tan(x)	Returns the tangent of an angle	YES

Table 13 List of JavaScript methods that are supported by solution

From the following list of methods, I have selected those that are most widely used and found appropriate LaTeX representation syntax. Additionally by adding operators and other required symbols this mapping table was created and is used in the application for parsing purposes.

Visual representation	**LaTeX syntax**	**JavaScript representation**
$x + y$	x + y	x + y
$x - y$	x - y	x - y
$\frac{x}{y}$	\frac{x}{y}	x / y
$x \cdot y$	x \cdot y	x * y
(x)	\left(x\right)	(x)
$\left\|x\right\|$	\left|x\right|	Math.abs(x)
$\operatorname{acos} x$	\acos x	Math.acos(x)
$\operatorname{asin} x$	\asin x	Math.asin(x)
$\operatorname{atan} x$	\atan x	Math.atan(x)
$\lceil x \rceil$	\lceil x\rceil	Math.ceil(x)

$\cos x$	\cos x	Math.cos(x)
$\lfloor x \rfloor$	\lfloor x\rfloor	Math.floor(x)
x^n	x^n	Math.pow(x,n)
$\sin x$	\sin x	Math.sin(x)
$\sqrt{x}$	\sqrt{x}	Math.sqrt(x)
$\tan x$	\tan x	Math.tan(x)

Table 14 LaTeX and JavaScript expressions mapping

By having this kind of mapping in place, parsing the LaTeX and converting it to JavaScript expression is fairly straightforward.

Visual representation	**LaTeX syntax**	**JavaScript representation**	**Evaluated value**
$12^{(2+2)}$	12^{\left(2+2\right)}	Math.pow(12, (((2 + 2))))	20736
$\sqrt{9}$	\sqrt[2]{9}	Math.pow(9, 1/(2)	3
$\frac{100}{20}$	\frac{100}{20}	(100/20)	5
$\lvert -3 \rvert$	\left|-3\right|	Math.abs(- 3)	3
$\lfloor 7.004 \rfloor$	\lfloor7.004\rfloor	Math.floor(7.004)	7
$\sin\frac{\pi}{2}$	\sin\frac{\pi}{2}	Math.sin(((3.141592653589793/2)) * 1)	1
$27^{\frac{1}{3}}$	27^\frac{1}{3}	Math.pow(27, ((1/3)))	3

Table 15 Examples of LaTeX syntax parsing and mapping to JavaScript methods

2.2.3.3 Using variables in LaTeX and substituting them with numbers

Sometimes parsing only text and calculating value is not enough. In some cases we don’t know all the values at the moment, but they will be known later. In this case we need to include a variable and substitute it later once it is known.

Consider the following expression:

$$5 + x^2$$

We can parse this expression, but we cannot evaluate it right away, since we need to find value of x first.

What solver does, when parsing this expression, is that it creates a list of variables that are required to be known in order to evaluate expression. Until all of them are known, expression cannot be evaluated. For the following expression only x is required.

Any single character a-z, A-Z or LaTeX symbol is considered a variable for the parser.

Visual representation	**LaTeX syntax**	**Found variables**
$12^{(x+y)}$	12^{\left(x+y\right)}	x , y
$\sqrt{x}$	\sqrt[2]{x}	x
$\frac{100}{x}$	\frac{100}{20}	x
$\lvert -x \rvert$	\left|-3\right|	x
$\lfloor x - 0.1 \rfloor$	\lfloor7.004\rfloor	x
$\sin\frac{\pi}{2}$	\sin\frac{\pi}{2}	π
$27^{\frac{y}{x}}$	27^\frac{1}{x}	x , y

Table 16 Example of LaTeX variable parsing results

Few issues occur when using variable parser; it cannot perform well in every possible case. Consider the following equation:

$$ax + by + c = 0$$

The parser will find correct variables in this case: a, b, x, y

But in this example, it will not perform correctly:

$$\frac{d}{dx}\sin x = \cos x$$

Since solver doesn’t know what derivative is, the variables list will consist of: d, x

The same problem can happen with geometric operations where distance between two points A and B is written as just AB. This is something that ordinary people just know from their experience, but for current solver, this is unsolved problem.

2.2.3.4 Solving model of multiple expressions or equations

For solving equations we need to find equality between elements. Typically equations are written in the form where every part separated by = is equal.

$$x = y = 2$$

If one part of the equation is found, then we know that it is equal to every other remaining part, so if $y = 2$ then we can find that $x = 2$

When the parser starts working, it will first break every equation into parts. Let’s take a look at the following example, and evaluate it:

1. Step: Initial model

   Blocks list:

$$x = y$$

$$x = 2 + 3b$$

$$b = z = 5$$

2. Step: Evaluate every block if it is possible, and remember evaluated value of a block

   Blocks list:

$x = y$	?
$x = 2 + 3b$	?
$b = z = 5$	5

3. Step: Create variables map, and remember values of every variable if it can be evaluated right now:

Blocks list:

$x = y$	?
$x = 2 + 3b$	?
$b = z = 5$	5

Variables map:

$$x = ?$$

$$y = ?$$

$$b = 5$$

$$z = 5$$

4. Step: Substitute variables in blocks with found values and evaluate blocks again:

Blocks list:

$x = y$	?
$x = 2 + 3 \cdot 5$	17
$5 = 5 = 5$	5

Variables map:

$$x = ?$$

$$y = ?$$

$$b = 5$$

$$z = 5$$

5. Step: Update variables map from results of blocks evaluation.

Blocks list:

$x = y$	?
$x = 2 + 3 \cdot 5$	17
$5 = 5 = 5$	5

Variables map:

$$x = 17$$

$$y = ?$$

$$b = 5$$

$$z = 5$$

6. Continue to Step 4, until there are no new variables evaluated and no blocks can be evaluated as well.

Final result:

Blocks list:

$x = y$	17
$x = 2 + 3 \cdot 5$	17
$5 = 5 = 5$	5

Variables map:

$$x = 17$$

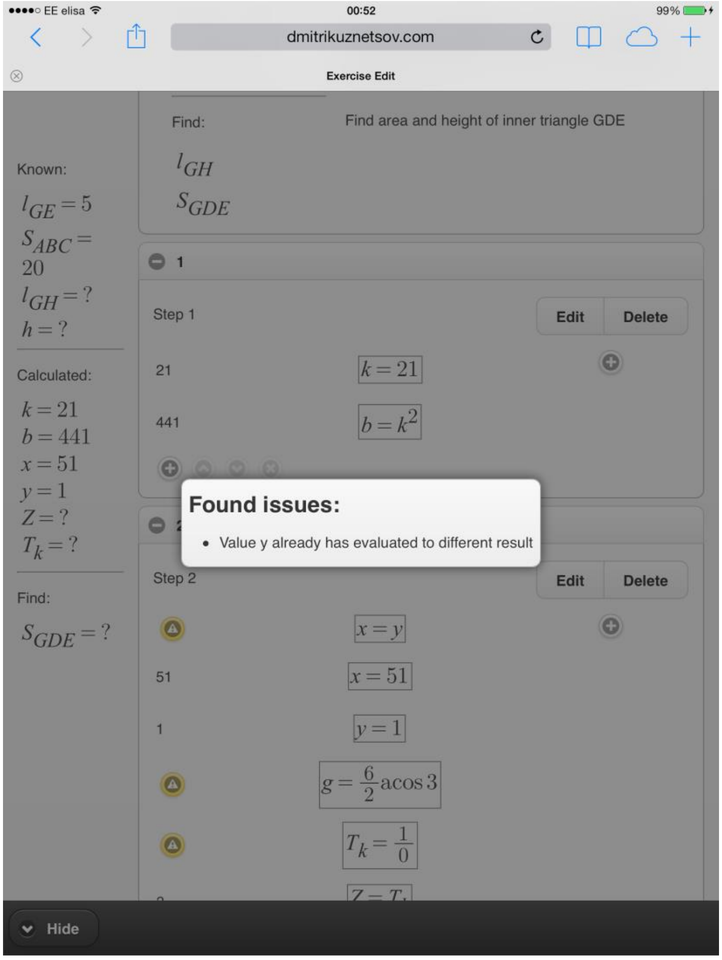
$$y = 17$$

$$b = 5$$

$$z = 5$$

This is fairly simple iterative process, but works very well for simple examples where user has to simplify equation manually in order to get it to the form that can be calculated.

As a side effect, of this approach, the model automatically verifies itself, so any errors will be quickly visible (the yellow error icon will be placed, next to the block where error occurred). By clicking on this icon, a popup will be opened revealing cause of the error.



Find: Find area and height of inner triangle GDE

l_{GH}

S_{GDE}

Known:

$l_{GE}=5$

$S_{ABC}=20$

$l_{GH}=?$

$h=?$

Calculated:

$k=21$

$b=441$

$x=51$

$y=1$

$Z=?$

$T_k=?$

Find:

$S_{GDE}=?$

Step 1

21 $k=21$

441 $b=k^2$

Found issues:

- Value y already has evaluated to different result

Step 2

$x=y$

51 $x=51$

1 $y=1$

$g=\frac{6}{2}\mathrm{acos}\,3$

$T_k=\frac{1}{0}$

Figure 44 Error message is displayed where evaluation has conflicted with already evaluated result

3 Summary

3.1 Results of the work

Primary result of this work is implemented solution – a browser-based program which promotes visual, model-based methods for solving problems in mathematics, physics, economics etc in intuitive and interactive way.

The following sections analyze results based on goals defined in section “Main goals of this work”,1.6 in more details.

3.1.1 Results for goal – “Create requirements for the program”

For the following goal, requirements were created in section “General requirements for program”, 1.7. These requirements were created in order to follow the same Teacher – Pupil relationship mode that is common in every school.

3.1.2 Results for goal – “Analyze other alternative solutions based on the requirements”

Based on the requirements that were created, other existing solutions were analyzed in section “Analysis of existing solutions”, 1.8. The main conclusion was that many existing solutions are quite flexible and powerful but doesn’t fit very well for educational work. It is too difficult to create exercises for the teacher, share them with pupils and control results. T-Algebra turned out to be the closest solution based on requirements, since it was created for educational purpose in the first place, but unfortunately it is only working on Windows computers.

3.1.3 Results for goal – “Implement visual solver application”

The implemented solution was created using client-server based architecture. This was required in order to support collaboration between teacher and pupils. For visualization of mathematical expressions MathQuill library was used.

Few problems have appeared during development process:

- Browser incompatibilities and different standard implementation:
    - Safari (webkit based browser) doesn't return correct element when button is clicked on (https://bugs.webkit.org/show_bug.cgi?id=22261). This feature was needed in order for ***Virtual Keyboard*** to work correctly. Alternative method was used instead.
    - Firefox - requires that event is passed for all the click events.
- JQuery mobile – AJAX implementation when navigating through pages doesn't allow refresh to be used, since scripts are loaded dynamically, to solve this problem a special script has to be added to every page that has to navigate user back to first screen.

The program is designed to be used in schools by pupils and teachers on mobile devices such as tablets and phones for their class- and homework. All of the requirements for this program that were specified in section 1.6 are fulfilled.

For the future, this program can be integrated with existing web based solutions or databases. For example list of teacher or pupils can be taken from another third-party software database.

Additional features could be added in the future:

- Not only teachers can add pictures, but pupils would be able to do so as well.
- Graphs can be added and automatically updated when model changes.
- Support for other types of exercises: simplification, finding critical points of a function, integration, etc.
- Better options for filtering out exercises, so exercises that were completed, will be hidden, to keep current list of exercise assignments clear.

3.2 Retrospective

Looking back at the beginning of development process, it is clear that decision to use Web technology was correct one. This gives a big edge over alternative solutions that are bound to specific platform, even if this application is not as feature reach, it is still available on platforms where alternatives just doesn't exist at all. New devices such as platforms are going to be used more and more in the future, and value of educational software for this platform will only grow in time. This is where VIPS does its job very well.

4 Kokkuvõte

VIPS – Visuaalne probleemide lahendaja

Magistritöö - Dmitri Kuznetsov

Selle töö peamiseks eesmärgiks oli luua veebipõhine programm, mis võimaldab kasutada visuaalseid modelleerimise meetodeid matemaatiliste, füüsikaliste, majanduslike jms probleemide lahendamiseks intuitiivsel ja interaktiivsel moel. Programm on peamiselt mõeldud kasutamiseks koolides või teistes asutustes õppimise eesmärgil. Selle tarkvara arendamiseks olid loodud nõudmised millele peaks taoline programm vastama.

Töö käigus analüüsiti olemasolevaid tarkvara lahendusi, nende omadusi (positiivsed, negatiivsed) ja tehti järeldused kas need programmid sobivad probleemi lahendamiseks, või mitte.

Selle töö lõpptulemusena loodi veebipõhine programm, mis annab õpetajatele ning õpilastele võimaluse omavahelist tööd paremini koordineerida ning lahendada ülesandeid kiiremini intuitiivsel ja tavapärasel moel. Programmi saab kasutada tahvelarvuti või muu seadme peal, millel on olemas interneti ühendus ja HTML5 toetusega veebilehitseja.

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6 Appendix

6.1 Database table structure definitions

In this section, detailed explanation of every field in solution class diagram is made.

User

Field name	Type	Description
ID	Integer	Unique ID of user
Login	String	Unique login name for the user
Name	String	Name of the user
HashedPassword	String	Password chosen by the user
IsPupil	Boolean	Indicates if user is a teacher or pupil

Session

Field name	Type	Description
ID	Integer	Unique ID of the session
UserID	[User]	User that is associated with this session
Login	Date	Time when login occurred
ExpireAt	Date	Indicates when the session will expire (typically +2 hours from first login, or last activity)
PHPSessionID	String	Session created by PHP

ExerciseTemplate

Field name	Type	Description
ID	Integer	Unique ID of the exercise template
TeacherID	[User]	Teacher who created this exercise template
Name	String	Short name of the exercise template
Description	String	Description
Created	Date	Time when template was created

ExerciseTemplateAssignment

Field name	Type	Description
ExerciseTemplateID	[ExerciseTemplate]	ID of the assignment
PupilID	[User]	Pupil who has this assignment
AssignedDate	Date	Time when assignment was made

ExerciseTemplateInfo

Field name	Type	Description
ID	Integer	Unique ID of the exercise template info
ExerciseTemplateID	[ExerciseTemplate]	ID of the exercise template this record belongs to
Latex	String	LaTeX expression to show to the user
IsKnown	Boolean	Indicates if this expression is known value of assignment (true), or (false) if it should be found.
LastModified	Date	Time when it was last modified

ExerciseTemplateDetails

Field name	Type	Description
ID	Integer	Unique ID of the exercise template details
ExerciseTemplateID	[ExerciseTemplate]	ID of the exercise template these details belong to
Description	String	Description of the exercise template (what is the scenario and what needs to be found in this assignment)
FileName	String	Original name of the uploaded image file
LocalPath	String	Location of the image on the server
LastModified	Date	Time when it was last modified

Exercise

Field name	Type	Description
ID	Integer	Unique ID of the exercise
ExerciseTemplateID	[ExerciseTemplate]	ID of the exercise template this exercise was created for
UserID	[User]	User who created this exercise
Created	Date	Time when this exercise was created
LastModified	Date	Time when it was last modified
PointsDeducted	Integer	Number of points that were deducted from pupil for using help tips
Mark	Integer	Final mark that was assigned by the teacher, -1 if no assignment has been made

ExerciseStrategy

Field name	Type	Description
ID	Integer	Unique ID of the exercise strategy
ExerciseID	[Exercise]	ID of the exercise this strategy was created for
Name	String	Name of the strategy
Description	String	Short description of the strategy
IsCompleted	Boolean	Indicates if user has marked this strategy as completed
IsCorrect	Boolean	If strategy was marked as completed, the server will verify if it is correct or not and mark this value automatically

ExerciseStep

Field name	Type	Description
ID	Integer	Unique ID of the exercise step
ExerciseStrategyID	[ExerciseStrategy]	ID of the exercise strategy this step belongs to
Description	String	Short description of the exercise step
LastModified	Date	Time when it was last modified

ExerciseStepUnlockedHelp

Field name	Type	Description
ID	Integer	Unique ID of the help element
ExerciseStepID	[ExerciseStep]	ID of the referenced exercise step
UnlockedEquations	Boolean	Indicates if referenced step equations were unlocked
UnlockedBlocks	Boolean	Indicates if referenced step blocks were unlocked

ExerciseStepBlockLatex

Field name	Type	Description
ID	Integer	Unique ID of this exercise step block
ExerciseStepID	[ExerciseStep]	ID of the referenced exercise step this block belongs to
Latex	String	LaTeX string that was added by the user
OrderNr	Integer	Indicates ordering of blocks. Blocks are order in ascending order.
LastModified	Date	Time when it was last modified

ExerciseStrategyCalculatedInfo

Field name	Type	Description
ExerciseStrategyID	[ExerciseStrategy]	ID of the exercise strategy this calculation result belongs to
ExerciseTemplateInfoID	[ExerciseTemplateInfo]	ID of the referenced template info this result is calculated for
EvaluationResult	Double	Result after calculation
HasEvaluated	Boolean	Indicates if calculation was successful
IsCorrect	Boolean	Indicates if calculation result is equal to the one found in exercise solution created by teacher. Always true if it is done by teacher.

ExerciseStepBlockEquation

Field name	Type	Description
ID	Integer	Unique ID of this exercise step block
ExerciseStepID	[ExerciseTemplateInfo]	ID of the referenced exercise step this block belongs to
EquationID	[Equation]	ID of the referenced equation
LastModified	Date	Time when it was last modified

Equation

Field name	Type	Description
ID	Integer	Unique ID of equation
Name	String	Name of the equation
Description	String	Short description of the equation
Latex	String	LaTeX text that will be shown to the user
UserID	[User]	Teacher who created this equation
Created	Date	Time when it was created

6.2 Calculation examples

6.2.1 Geometry – calculating area of inner triangle

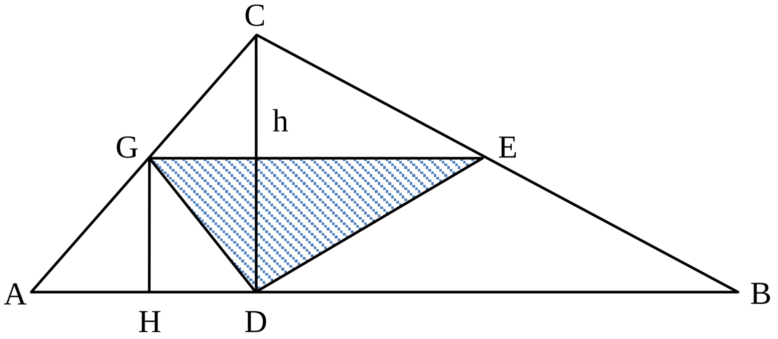
Total area of triangle is $20\ cm^2$. Inside the triangle ABC another triangle is drawn GDE, with the upper edge GE cutting triangle ABC in half. Length of the cut is $5\ cm$. Find area and height of inner triangle [36](Exercise 1, page 62).

Given:

$$GE = 5\ cm$$

$$S_{ABC} = 20\ cm^2$$

$$GH = \frac{h}{2}$$



Find:

$$GH, S_{GDE}$$

Solution:

Find the length of the bottom line AB, knowing that GE cuts triangle in half we can find it:

$$AB = 2GE = 10\ cm$$

1. Let’s find the height of the triangle:

$$S_{ABC} = \frac{1}{2} AB \ \cdot h$$

$$h = \frac{2S_{ABC}}{AB}$$

$$h = \frac{2 \ \cdot 20}{10} = 4 \ cm$$

2. Find the height of inner triangle

$$GH = \frac{h}{2} = \frac{4}{2} = 2 \ cm$$

3. Find the area of inner triangle

$$S_{GDE} = \frac{1}{2} GE \ \cdot GH$$

$$S_{GDE} = \frac{5 \ \cdot 2}{2} = 5 \ cm^2$$

Answer:

Height of inner triangle is 2 cm and area 5 cm^2

6.2.2 Geometry – calculating area of a triangle in circle

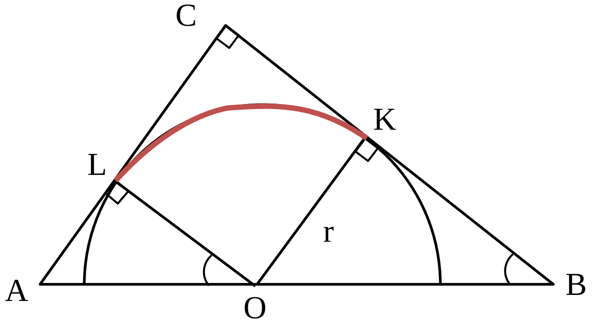
A triangle with right angles is drawn into half circle that intercepts another outer triangle sides. The diameter of half circle is located on the outer triangle hypotenuse and center of the half circle is dividing it by two sides 15 and 20. Find the length of the arc between the intersection points [37](Exercise 4, page 63).

Given:

$AO = \ 15$

$OB = 20$

$\angle AOL \approx \ \angle OBK$



Find:

LK

Solution:

1. Declare radius of half circle

$$OL = OK = r$$

2. Because square has a right angle $\angle LCK = 90^o$ then it is a square, so we can declare

$$\angle CLO = \angle CKO = 90^o$$

3. Let's find the radius r. Due to the fact that $\angle AOL \approx \angle OBK$, we can find the relation

$$\frac{AL}{AO} = \frac{OK}{OB}$$

$$\varDelta AOL : AL = \sqrt{AO^2 - OL^2} = \sqrt{15^2 - r^2}$$

, continue by adding values and simplifying

$$\frac{\sqrt{15^2 - r^2}}{15} = \frac{r}{20}$$

$$\sqrt{15^2 - r^2} = \frac{3}{4}r$$

$$15^2 - r^2 = \frac{9}{16}r^2$$

$$\frac{25}{16}r^2 = 15^2$$

$$r = \sqrt{\frac{15^2 \cdot 16}{25}} = \sqrt{\frac{3600}{25}} = \sqrt{144} = 12$$

4. The required arc LK both intersection points are created with angle 90^o from half circle center. This means that the actual arc length is quarter of the circle length, or

$$LK = \frac{1}{4} \cdot 2\pi r = \frac{1}{4} \cdot 2\pi \cdot 12 = 6\pi$$

Answer:

Length of the arc between the intersection points is 6π